The number of pancyclic arcs in a k-strong tournament

Anders Yeo
Department of Computer Science, Royal Holloway,University of London
Egham Surrey, TW20 0EX United Kingdom

A tournament is a digraph, where there is precisely one arc between every pair of distinct vertices. An arc is pancyclic in a digraph D, if it belongs to a cycle of length l, for all $3 <= l <= |V(D)|$. Let $p(D)$ denote the number of pancyclic arcs in a digraph D and let $h(D)$ denote the maximum number of pancyclic arcs belonging to the same Hamilton cycle of D. Note that $p(D) >= h(D)$. Moon showed that $h(T) >= 3$ for all strong non-trivial tournaments, T, and Havet showed that $h(T) >= 5$ for all 2-strong tournaments T. We will in this talk show that if T is a k-strong tournament, with $k >= 2$, then $p(T) >= nk/2$ and $h(T) >= (k + 5)/2$. This solves a conjecture by Havet, stating that there exists a constant a_k, such that $p(T) >= a_k * n$, for all k-strong tournaments, T, with $k >= 2$. Furthermore the second result gives support for the conjecture $h(T) >= 2k + 1$, which was also stated by Havet. The previously best known bounds when $k >= 2$ were $p(T) >= 2k + 3$ and $h(T) >= 5$. Furthermore some of the lemma's used in the above proofs immediately imply that every regular tournament is arc-pancyclic (which was first proved by Alspach), and that every 2-strong tournament contains 2 distinct vertices, such that all arcs out of them are arc-pancyclic. We conjecture that there in fact exists 3 such vertices, which would be best possible (even if we looked at k-strong tournaments for any fixed $k > 1$).