Subgraphs in vertex neighbourhoods of K_r-free graphs

Stephan Brandt

TU Ilmenau

The neighbourhood of every vertex of a triangle-free graph forms an independent set. In 1981, Janos Pach characterized those triangle-free graphs where every independent set belongs to the neighbourhood of a vertex. I will present alternative characterizations and indicate some applications of this result.

There are two natural ways to generalize this problem to K_r-free graphs: Characterize those K_r-free graphs where

(1) every independent set belongs to the neighbourhood intersection of a subgraph K_{r-2}, or

(2) every K_{r-1}-free subgraph belongs to the neighbourhood of a vertex.

It is not difficult to verify that property (2) implies property (1). Jointly with Jørgen Bang-Jensen we characterized the K_r-free graphs with property (2), based on Pach's result. The characterization of K_r-free graphs with property (1) is open even in the case $r = 4$.