On the diameter of the transportation polytope

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The transportation problem (TP) is a classic problem in operations research. The $m \times n$ TP has m supply points and n demand points. Each supply point i holds a quantity $r_i > 0$, and each demand point j wants a quantity $c_j > 0$. The total supply is equal to the total demand, hence $\sum_{i=1}^{m} r_i = \sum_{j=1}^{n} c_j$. Suppose we must transport non-negative quantities x_{ij} from nodes i to j so that all demands are met. The collection of all feasible solutions forms a convex polytope, the *transportation polytope* $\mathcal{T}$.

We are interested in the diameter of the skeleton (the edge-graph) of $\mathcal{T}$. In the talk we give some motivation why this is an interesting problem.

We also show how the problem to find the diameter can be formulated completely in terms of operations on subgraphs of edge-weighted complete bipartite graphs. This allows for a fairly easy proof of a linear bound on the diameter : $diam(\mathcal{T}) \leq 8\,(m + n)$.

This is joint work with Graham Brightwell (London School of Economics) and Leen Stougie (Technische Universiteit Eindhoven and CWI Amsterdam, the Netherlands).