Weakly four-connected graphs, two-connected orientations, and removable subgraphs

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We shall summarize a number of recent results obtained while attacking the problem of finding highly vertex-connected orientations of graphs. The characterization of graphs with k-edge-connected orientations follows immediately from a result of Nash-Williams from 1960. However, the vertex-connected case is still open for $k \geq 2$.

A more modest goal would be to verify a conjecture of Thomassen: for every k there exists an $f(k)$ such that every $f(k)$-vertex-connected graph has a k-vertex-connected orientation. This conjecture from 1985 has also been open, even for $k = 2$. We shall sketch the proof of this conjecture for $k = 2$ and show how the proof methods inspired other new results on weakly four-connected graphs as well as on removable cycles and trees, and some new algorithmic observations.