Obligatory Problems 2012

M M 508

The Problems should be handed in either to me or to the secretary not later than December 27., 2012 at 12 noon.
The problems consist of the one below and some of the earlier exam problems.

**OPGAVE**

Let \((X, d)\) be a metric space.

(i) Let \(x, y, x_1, y_1 \in X\). Prove that
\[
d(x, y) - d(x_1, y_1) \leq d(x, x_1) + d(y, y_1)
\]
and show next that
\[
|d(x, y) - d(x_1, y_1)| \leq d(x, x_1) + d(y, y_1).
\]

(ii) Use equation (2) to prove that \(d\) is a uniformly continuous function from \(X \times X\) to \([0, \infty[\) when \(X \times X\) is equipped with one of the equivalent metrics defined on page 47 of the notes.

**Earlier exam questions:**

- January 2006: Problem 1: 1, 2, 3, 5 (determine only \(int(S)\)).
- June 2011: Problem 2: 1, 3.