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GRAPH THEORY COLLOQUIUM

Domination and colorings

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Abstract:

I will present results on tight regular tournaments using the structure of its domination digraph.

The *domination graph* of a tournament T is the graph on the vertex set $V(T)$ with edges between dominant pairs of vertices of T. The *domination digraph* $\mathfrak{D}(T)$ of a tournament T is the domination graph of T with the orientation induced by T. The domination digraph was introduced and studied by Fisher et. al. in [4] and the domination graphs of regular tournaments was characterized by Cho et al. in [2, 3].

A tournament T is said to be *tight* if for every coloring of $V(T)$ with exactly 3 colors, there exists a heterochromatic directed triangle. This definition is related to the $\overrightarrow{C}_3$*-free disconnection* of a digraph D introduced by V. Neumann-Lara in [9] and the *heterochromatic number* of a 3-*graph* defined in [1]. The $\overrightarrow{C}_3$-free disconnection $\overrightarrow{\omega}_3(D)$ of a digraph D is the maximum number of colors r such that for every coloring of $V(D)$ with exactly r colors, there is no heterochromatic directed triangle. Thus, a tournament T is tight if and only if $\overrightarrow{\omega}_3(T) = 2$. Infinite families of tight circulant tournaments are given in [5, 6, 9].

A regular tournament T is called an *ample tournament* if every maximal path in $\mathfrak{D}(T)$ has order at least 3. If every directed path in $\mathfrak{D}(T)$ has order at most 2, then T is called a *mold* [11]. The concept of molds breaks regular tournaments naturally into three classes: cyclic tournaments, tame tournaments and wild tournaments. The cyclic tournament $\overrightarrow{C}_{2m+1}\langle\emptyset\rangle$, $m \geq 1$, is the tournament on the vertex set $\mathbb{Z}_{2m+1}$ and $uv \in A\left(\overrightarrow{C}_{2m+1}\langle\emptyset\rangle\right)$ if $v - u \in \mathbb{Z}_m \setminus \{0\}$. A regular tournament is said to be *tame* if it has a maximal path P in $\mathfrak{D}(T)$ such that the residue of P is isomorphic to a cyclic tournament. Note that $\mathfrak{D}(T)$ is a spanning cycle if $T \cong \overrightarrow{C}_{2m+1}\langle\emptyset\rangle$ and $\mathfrak{D}(T)$ is a disjoint set of at least 4 directed paths, whenever $T \ncong \overrightarrow{C}_{2m+1}\langle\emptyset\rangle$.

We prove that the two classes of regular tournaments, tame tournaments and ample tournaments are tight. Moreover, any tournament with a tight mold is also tight [7]. Finally, we prove that the dichromatic number of a tame tournament is 3 [10]. The *dichromatic number* of a digraph D was defined by V. Neumann-Lara in [8], as the minimum number of colors needed to color the vertices of D such a way that each chromatic class is acyclic.

References

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Host: Jørgen Bang-Jensen