Drawing Rotations

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cyclic permutation of neighbors at a vertex is called a local rotation

K_4 in R^2

G

Collection of local rotations for each vertex is a rotation (scheme)

1: (432)
2: (431)
3: (421)
4: (321)

Drawing avoids

a) \( \equiv \) (self-crossing)
b) \( \times \times \) (multiple-crossing)
c) \( \circlearrowleft \) (adjacent-crossing)

Lemma: Drawing \( \Rightarrow \) Rotation
Question: Which rotations correspond to drawings

Example: 

\begin{align*}
\begin{array}{cccc}
1 & 2 & 1: (432) \\
4 & 3 & 2: (431) \\
& & 3: (421) \\
& & 4: (123)
\end{array}
\end{align*}

Motivation: From drawing of $K_n$ get rotation, & from rotation can count $\#$ crossings in drawing

Knowing which rotations come from drawings might help get lower bound on $cr(K_n)$

Might even be possible min "#bad $K_u" is conjectured lower bound for $cr(K_n)$ where min is over all rotations 
bad $K_u$ is drawable crossing or non-drawable $K_u$