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Cycle Spectra

Def: **cycle spectrum** = set of cycle lengths
pancyclic = spectrum $\{3, \ldots, n\}$

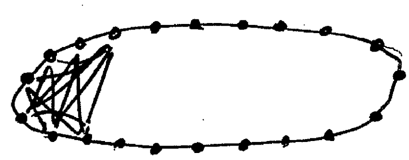
Bondy's Metaconjecture [1971]: sufficient conditions for Hamiltonian cycles tend to also imply pancyclic

Jacobson-Lehel [1999]: How small can the spectrum of a k-regular Hamiltonian graph be?

previous lower bound: $c \log n$

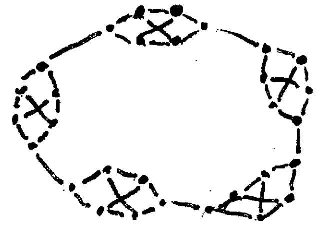
Theorem (Milans-West) If G is Hamiltonian, has n vertices, has average degree d, then G has at least $\sqrt{\frac{n(d-2)}{2\lg[n(d-2)]}}$ distinct cycle lengths.

sharpness:



$2\sqrt{n(d-2)}$ lengths

Regular graphs: known upper bound is linear



$|\text{spectrum}| = \frac{n}{6} + 3$

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Arboricity Ratio

Def: $\Upsilon(G)$ = min #forests w. union G - arboricity
$\Theta(G)$ = min #planar graphs w. union G - thickness

$\Upsilon(G) \le 3$ if G is planar $\Longrightarrow$ $1 \le \dfrac{\Upsilon(G)}{\Theta(G)} \le 3$
≤ 2 if G is planar & triangle-free . . . . . ≤ 2

Ex: $\Theta(K_n) = \left\lceil \dfrac{\binom{n}{2}}{3n-6} \right\rceil = \left\lceil \dfrac{n+1}{6} \right\rceil$ $\quad \Upsilon(K_n) = \left\lceil \dfrac{n}{2} \right\rceil$ ratio ≈ 3

$\Theta(K_{p,p}) = \left\lceil \dfrac{p^2}{4p-4} \right\rceil = \left\lceil \dfrac{p+2}{4} \right\rceil$ $\quad \Upsilon(K_{p,p}) = \left\lceil \dfrac{p+1}{2} \right\rceil$ ratio ≈ 2

Ques: $\delta(G)$ large $\Longrightarrow$ $r(G)$ large ?
$\delta(G) \ge n/2 \Longrightarrow r(G) \ge 2 - o(n^{-1})$?
$\delta(G) \ge 2n/3 \Longrightarrow r(G) \ge 3 - o(n^{-1})$?