Computational complexity of some colouring problems

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Abstract

A favourable result in computational complexity for a hierarchy of problems is a dichotomy result, a classification of the problems into two branches, the tractable and the intractable ones. For instance Schaefers classical dichotomy result for the hierarchy of generalized satisfiability problems classifies the problems into polynomial time solvable and NP-complete ones. Motivated by the result that the 3-colourability problem remains NP-complete for the class of graphs with maximum degree four and the linear time solvability for the family of graphs having maximum degree three due to the celebrated theorem of Brooks we study a family of intermediate graph classes between theses two extremes in order to make the complexity gap more precise. For this problem hierarchy we are able to classify the problems into NP-complete and linear time solvable ones, i. e. dichotomy holds.

In this talk we also discuss the computational complexity of determining the chromatic number of graphs without long induced paths. By usage of the data structure 'Cotree' for representing P_4-free graphs it is not very difficult to obtain a linear time algorithm for deciding, whether a P_4-free graph is l-colourable for $l \in N$. For the cases $k = 5, 6$ we present polynomial time algorithms for deciding, whether a P_k-free graph is 3-colourable. Moreover, these algorithms can be used to l-colour the corresponding YES-instances of the decision problems. For the families of P_5-free graphs and for P_6-free graphs our approach is based on an encoding of the problem with boolean formulas making use of the existence of bounded dominating subgraphs. Especially, we need a structural analysis of the non-perfect K_4-free members of the class of P_6-free 3-colourable graphs. Furthermore, we show the NP-completeness of deciding whether a P_8-free graph is 5-colourable and of deciding whether a P_{12}-free graph is 4-colourable due to Sgall and Woeginger. An interesting byproduct is a structural analysis of the family of non-3-colourable, P_6-free and triangle-free graphs. Basically, every member of this family is 4-chromatic, contains the Mycielski-Grötzsch graph as induced subgraph and is the induced subgraph of the Clebsch graph.