Computer Proofs

Luís Cruz-Filipe

Mathematics throughout the ages

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- finding solutions for practical problems

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$$\exists x . ax^3 + bx^2 + cx + d = 0$$

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- able to perform very complicated computations

...but *pure* mathematics?

- computer as calculator

- often little semantics, no mathematical concepts

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"Teach" the computer as a human:

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In return we get:

- higher degree of confidence in the mathematics

- better reliability as a calculator

$$e^{i\pi} + 1 = 0$$

Euler, 1748

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$$\mathrm{Exp}(i \times \pi) + 1 = 0$$

Coq, 2003