C-CoRN: The Constructive Coq Repository @ Nijmegen

Dutch Proof Tools Day, 9 July 2004

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The Coq Repository @ Nijmegen

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1. Overview of CoRN and C-CoRN

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2. History

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5. Future Directions

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What?

Where?

Why?

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Analyze the process of formalizing mathematics

The FTA project

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People: H. Barendregt, H. Geuvers, M. Niqui, R. Pollack, F. Wiedijk, J. Zwanenburg

Real Analysis & C-CoRN

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Library of transcendental functions.

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- Applications: algebraic reasoning, program extraction

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  - focus on mathematical and metaformalization issues

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  - trigonometry: $\forall_{x:\mathbb{R}}.\cos^2(x) + \sin^2(x) = 1$
  - complex numbers: $e^{i\pi} + 1 = 0$

Examples (cont.)

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approximation	value of e
0	$\frac{0}{1} = 0$
1	$\frac{1}{1} = 1$
2	$\frac{2}{1} = 2$
5	$\frac{65}{24} \approx 2.70833$
10	$\frac{98641}{36288} \approx 2.71828$

Examples (cont.)

- program extraction: computed values of constants

approximation	value of $\sqrt{2}$
0	$\frac{0}{1} = 0$
1	$\frac{3}{3} = 1$
2	$\frac{3}{3} = 1$
5	$\frac{35}{27} \approx 1.2963$
10	$\frac{27755}{19683} \approx 1.4101$

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