First-Order Logic with Domain Conditions

Logic and Computation Seminar, 23 July 2004

Luís Cruz-Filipe

(joint work with Herman Geuvers & Freek Wiedijk)

University of Nijmegen, Netherlands

Center for Logic and Computation, Portugal

Contents

Contents

Contents

Contents

Contents

Contents

Motivation

Motivation

- no adequate treatment of partiality in logic

Motivation

- no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)

Motivation

- no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL

Motivation

- no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL
  - no semantics

Motivation

- no adequate treatment of partiality in logic
- paper by Wiedijk and Zwanenburg (TPHOLs 2003)
  - syntactic system, equivalent to FOL
  - no semantics
  - overpermissive presentation of FOL

The three systems

Three equivalences:

The three systems

Three equivalences:

- D $\leftrightarrow$ P: easy, uninteresting

The three systems

Three equivalences:

- D $\leftrightarrow$ P: easy, uninteresting
- D $\leftrightarrow$ T: syntactic level on the paper

$$\mathcal{DC}(\Gamma), \mathcal{DC}_\Gamma(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

The three systems

Three equivalences:

- $\mathsf{D} \leftrightarrow \mathsf{P}$: easy, uninteresting
- $\mathsf{D} \leftrightarrow \mathsf{T}$: syntactic level on the paper

  $$\mathcal{DC}(\Gamma), \mathcal{DC}_\Gamma(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

  semantics new

The three systems

Three equivalences:

- D $\leftrightarrow$ P: easy, uninteresting
- D $\leftrightarrow$ T: syntactic level on the paper

  $$\mathcal{DC}(\Gamma), \mathcal{DC}_\Gamma(\varphi), \Gamma \vdash^{\mathsf{T}} \varphi \text{ iff } \Gamma \vdash^{\mathsf{D}} \varphi$$

  semantics new
- T $\leftrightarrow$ FOL: not trivial, not done on the paper

Derivation Rules for FOL

$$(\textit{assum})\ \frac{}{\Gamma \vdash^{\mathsf{FOL}} \varphi}\ \varphi \in \Gamma \qquad (\neg\neg\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{FOL}} \neg\neg\varphi}{\Gamma \vdash^{\mathsf{FOL}} \varphi}$$

$$(\rightarrow\text{-}I)\ \frac{\Gamma, \varphi \vdash^{\mathsf{FOL}} \psi}{\Gamma \vdash^{\mathsf{FOL}} (\varphi \rightarrow \psi)} \qquad (\rightarrow\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{FOL}} (\varphi \rightarrow \psi) \quad \Gamma \vdash^{\mathsf{FOL}} \varphi}{\Gamma \vdash^{\mathsf{FOL}} \psi}$$

$$(\forall\text{-}I)\ \frac{\Gamma \vdash^{\mathsf{FOL}} \varphi}{\Gamma \vdash^{\mathsf{FOL}} (\forall x_i.\, \varphi)}\ x_i \notin FV(\Gamma) \qquad (\forall\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{FOL}} (\forall x_i.\, \varphi)}{\Gamma \vdash^{\mathsf{FOL}} \varphi[x_i := t]}\ *$$

$$(\textit{refl})\ \frac{}{\Gamma \vdash^{\mathsf{FOL}} t = t} \qquad (\textit{sym})\ \frac{\Gamma \vdash^{\mathsf{FOL}} t = t'}{\Gamma \vdash^{\mathsf{FOL}} t' = t} \qquad (\textit{trans})\ \frac{\Gamma \vdash^{\mathsf{FOL}} t_1 = t_2 \quad \Gamma \vdash^{\mathsf{FOL}} t_2 = t_3}{\Gamma \vdash^{\mathsf{FOL}} t_1 = t_3}$$

$$(=\text{-}\textit{fun})\ \frac{\Gamma \vdash^{\mathsf{FOL}} t_1 = t'_1 \quad \cdots \quad \Gamma \vdash^{\mathsf{FOL}} t_{a_i} = t'_{a_i}}{\Gamma \vdash^{\mathsf{FOL}} f_i(t_1, \ldots, t_{a_i}) = f_i(t'_1, \ldots, t'_{a_i})}$$

$$(=\text{-}\textit{pred})\ \frac{\Gamma \vdash^{\mathsf{FOL}} t_1 = t'_1 \quad \cdots \quad \Gamma \vdash^{\mathsf{FOL}} t_{r_i} = t'_{r_i}}{\Gamma \vdash^{\mathsf{FOL}} P_i(t_1, \ldots, t_{r_i}) \rightarrow P_i(t'_1, \ldots, t'_{r_i})}$$

Derivation Rules for T – I

$$(\epsilon\text{-}\textit{wf})\ \frac{}{\epsilon \vdash^{\mathsf{T}} \textit{wf}} \quad (\textit{decl-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma, x_i \vdash^{\mathsf{T}} \textit{wf}} \quad (\textit{assum-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \varphi\ \textit{wf}}{\Gamma, \varphi \vdash^{\mathsf{T}} \textit{wf}}$$

$$(\textit{var-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} x_i\ \textit{wf}}\ x_i \in \Gamma \quad (\textit{const-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} c_i\ \textit{wf}}$$

$$(\textit{fun-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i}\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} f_i(t_1, \ldots, t_{a_i})\ \textit{wf}}$$

$$(\textit{if-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \vartheta\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2)\ \textit{wf}}$$

$$(\bot\text{-}\textit{wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} \bot\ \textit{wf}} \quad (\rightarrow\text{-}\textit{wf})\ \frac{\Gamma \vdash^{\mathsf{T}} \varphi\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} \psi\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} (\varphi \rightarrow \psi)\ \textit{wf}} \quad (\forall\text{-}\textit{wf})\ \frac{\Gamma, x_i \vdash^{\mathsf{T}} \varphi\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} (\forall x_i.\, \varphi)\ \textit{wf}}$$

$$(=\text{-}\textit{wf})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} t_1 = t_2\ \textit{wf}} \quad (\textit{pred-wf})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{r_i}\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} P_i(t_1, \ldots, t_{r_i})\ \textit{wf}}$$

Derivation Rules for T – II

$$(\textit{assum})\ \frac{\Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} \varphi}\ \varphi \in \Gamma \qquad (\to\text{-}I)\ \frac{\Gamma, \varphi \vdash^{\mathsf{T}} \psi}{\Gamma \vdash^{\mathsf{T}} (\varphi \to \psi)} \qquad (\to\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{T}} (\varphi \to \psi) \quad \Gamma \vdash^{\mathsf{T}} \varphi}{\Gamma \vdash^{\mathsf{T}} \psi}$$

$$(\neg\neg\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{T}} \neg\neg\varphi}{\Gamma \vdash^{\mathsf{T}} \varphi} \qquad (\forall\text{-}I)\ \frac{\Gamma, x_i \vdash^{\mathsf{T}} \varphi}{\Gamma \vdash^{\mathsf{T}} (\forall x_i.\, \varphi)} \qquad (\forall\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{T}} (\forall x_i.\, \varphi) \quad \Gamma \vdash^{\mathsf{T}} t\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} \varphi[x_i := t]}$$

$$(\textit{refl})\ \frac{\Gamma \vdash^{\mathsf{T}} t\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} t = t} \qquad (\textit{sym})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_2}{\Gamma \vdash^{\mathsf{T}} t_2 = t_1} \qquad (\textit{trans})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t_2 \quad \Gamma \vdash^{\mathsf{T}} t_2 = t_3}{\Gamma \vdash^{\mathsf{T}} t_1 = t_3}$$

$$(=\text{-}\textit{fun})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t'_1 \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{a_i} = t'_{a_i} \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} f_i(t_1, \ldots, t_{a_i}) = f_i(t'_1, \ldots, t'_{a_i})}$$

$$(=\text{-}\textit{pred})\ \frac{\Gamma \vdash^{\mathsf{T}} t_1 = t'_1 \quad \cdots \quad \Gamma \vdash^{\mathsf{T}} t_{r_i} = t'_{r_i} \quad \Gamma \vdash^{\mathsf{T}} \textit{wf}}{\Gamma \vdash^{\mathsf{T}} P_i(t_1, \ldots, t_{r_i}) \to P_i(t'_1, \ldots, t'_{r_i})}$$

$$(=\text{-}\textit{if-true})\ \frac{\Gamma \vdash^{\mathsf{T}} \vartheta \quad \Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) = t_1}$$

$$(=\text{-}\textit{if-false})\ \frac{\Gamma \vdash^{\mathsf{T}} \neg\vartheta \quad \Gamma \vdash^{\mathsf{T}} t_1\ \textit{wf} \quad \Gamma \vdash^{\mathsf{T}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{T}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) = t_2}$$

Derivation Rules for D – I

$$(\epsilon\text{-}\mathit{wf})\ \frac{}{\epsilon \vdash^{\mathsf{D}} \mathit{wf}} \qquad (\mathit{decl\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} \mathit{wf}}{\Gamma, x_i \vdash^{\mathsf{D}} \mathit{wf}} \qquad (\mathit{assum\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} \varphi\ \mathit{wf}}{\Gamma, \varphi \vdash^{\mathsf{D}} \mathit{wf}}$$

$$(\mathit{var\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} x_i\ \mathit{wf}}\ x_i \in \Gamma \qquad (\mathit{const\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} c_i\ \mathit{wf}}$$

$$(\mathit{fun\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} D_{f_i}(t_1, \ldots, t_{a_i})}{\Gamma \vdash^{\mathsf{D}} f_i(t_1, \ldots, t_{a_i})\ \mathit{wf}}$$

$$(\mathit{if\text{-}wf})\ \frac{\Gamma, \vartheta \vdash^{\mathsf{D}} t_1\ \mathit{wf} \quad \Gamma, \neg\vartheta \vdash^{\mathsf{D}} t_2\ \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2)\ \mathit{wf}}$$

$$(\bot\text{-}\mathit{wf})\ \frac{\Gamma \vdash^{\mathsf{D}} \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} \bot\ \mathit{wf}} \qquad (\rightarrow\text{-}\mathit{wf})\ \frac{\Gamma, \varphi \vdash^{\mathsf{D}} \psi\ \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} (\varphi \rightarrow \psi)\ \mathit{wf}} \qquad (\forall\text{-}\mathit{wf})\ \frac{\Gamma, x_i \vdash^{\mathsf{D}} \varphi\ \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} (\forall x_i.\, \varphi)\ \mathit{wf}}$$

$$(=\text{-}\mathit{wf})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1\ \mathit{wf} \quad \Gamma \vdash^{\mathsf{D}} t_2\ \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} t_1 = t_2\ \mathit{wf}} \qquad (\mathit{pred\text{-}wf})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1\ \mathit{wf} \quad \cdots \quad \Gamma \vdash^{\mathsf{D}} t_{r_i}\ \mathit{wf} \quad \Gamma \vdash^{\mathsf{D}} \mathit{wf}}{\Gamma \vdash^{\mathsf{D}} P_i(t_1, \ldots, t_{r_i})\ \mathit{wf}}$$

Derivation Rules for D – II

$$(\textit{assum})\ \frac{\Gamma \vdash^{\mathsf{D}} \textit{wf}}{\Gamma \vdash^{\mathsf{D}} \varphi}\ \varphi \in \Gamma \qquad (\rightarrow\text{-}I)\ \frac{\Gamma, \varphi \vdash^{\mathsf{D}} \psi}{\Gamma \vdash^{\mathsf{D}} (\varphi \rightarrow \psi)} \qquad (\rightarrow\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{D}} (\varphi \rightarrow \psi) \quad \Gamma \vdash^{\mathsf{D}} \varphi}{\Gamma \vdash^{\mathsf{D}} \psi}$$

$$(\neg\neg\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{D}} \neg\neg\varphi}{\Gamma \vdash^{\mathsf{D}} \varphi} \qquad (\forall\text{-}I)\ \frac{\Gamma, x_i \vdash^{\mathsf{D}} \varphi}{\Gamma \vdash^{\mathsf{D}} (\forall x_i.\, \varphi)} \qquad (\forall\text{-}E)\ \frac{\Gamma \vdash^{\mathsf{D}} (\forall x_i.\, \varphi) \quad \Gamma \vdash^{\mathsf{D}} t\ \textit{wf}}{\Gamma \vdash^{\mathsf{D}} \varphi[x_i := t]}$$

$$(\textit{refl})\ \frac{\Gamma \vdash^{\mathsf{D}} t\ \textit{wf}}{\Gamma \vdash^{\mathsf{D}} t = t} \qquad (\textit{sym})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_2}{\Gamma \vdash^{\mathsf{D}} t_2 = t_1} \qquad (\textit{trans})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t_2 \quad \Gamma \vdash^{\mathsf{D}} t_2 = t_3}{\Gamma \vdash^{\mathsf{D}} t_1 = t_3}$$

$$(=\text{-}\textit{fun})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t'_1 \cdots \Gamma \vdash^{\mathsf{D}} t_{a_i} = t'_{a_i} \quad \Gamma \vdash^{\mathsf{D}} D_{f_i}(t_1, \ldots, t_{a_i}) \quad \Gamma \vdash^{\mathsf{D}} D_{f_i}(t'_1, \ldots, t'_{a_i})}{\Gamma \vdash^{\mathsf{D}} f_i(t_1, \ldots, t_{a_i}) = f_i(t'_1, \ldots, t'_{a_i})}$$

$$(=\text{-}\textit{pred})\ \frac{\Gamma \vdash^{\mathsf{D}} t_1 = t'_1 \quad \cdots \quad \Gamma \vdash^{\mathsf{D}} t_{r_i} = t'_{r_i} \quad \Gamma \vdash^{\mathsf{D}} \textit{wf}}{\Gamma \vdash^{\mathsf{D}} P_i(t_1, \ldots, t_{r_i}) \rightarrow P_i(t'_1, \ldots, t'_{r_i})}$$

$$(=\text{-}\textit{if-true})\ \frac{\Gamma \vdash^{\mathsf{D}} \vartheta \quad \Gamma, \vartheta \vdash^{\mathsf{D}} t_1\ \textit{wf} \quad \Gamma, \neg\vartheta \vdash^{\mathsf{D}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{D}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) = t_1}$$

$$(=\text{-}\textit{if-false})\ \frac{\Gamma \vdash^{\mathsf{D}} \neg\vartheta \quad \Gamma, \vartheta \vdash^{\mathsf{D}} t_1\ \textit{wf} \quad \Gamma, \neg\vartheta \vdash^{\mathsf{D}} t_2\ \textit{wf}}{\Gamma \vdash^{\mathsf{D}} (\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) = t_2}$$

Semantics of $\mathcal{T}$ – I

Semantics of T – I

- T-models are FOL-models

Semantics of T – I

- T-models are FOL-models
- a T-substitution for $\mathfrak{M}$ is a partial function that assigns values in A to some variables x_i

Semantics of T *– I*

- T-models are FOL-models
- a T-substitution for $\mathfrak{M}$ is a partial function that assigns values in A to some variables x_i
- $[\![t]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}$, $\models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf*, $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* and $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ defined simultaneously

Semantics of T – I

- T-models are FOL-models
- a T-substitution for $\mathfrak{M}$ is a partial function that assigns values in A to some variables x_i
- $[\![t]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}$, $\models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf*, $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* and $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ defined simultaneously

$$
\begin{aligned}
[\![x_i]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} &:= \rho(x_i) \\
[\![c_i]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} &:= [\![c_i]\!]^{\mathsf{T}}_{\mathfrak{M}} \\
[\![f_i(t_1,\ldots,t_{a_i})]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} &:= [\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}}\left([\![t_1]\!]^{\mathsf{T}}_{\mathfrak{M},\rho},\ldots,[\![t_{a_i}]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}\right) \\
[\![\text{if } \vartheta \text{ then } t_1 \text{ else } t_2]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} &:= \begin{cases} [\![t_1]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} & \text{if } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \vartheta \\ [\![t_2]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} & \text{if } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \neg\vartheta \end{cases}
\end{aligned}
$$

Semantics of T – II

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} \bot \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} P_i(t_1, \ldots, t_{r_i}) \textit{ wf} \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 \textit{ wf}, \ldots, \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_{r_i} \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 = t_2 \textit{ wf} \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 \textit{ wf} \text{ and } \models^{\mathsf{T}}_{\mathfrak{M},\rho} t_2 \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} (\varphi \to \psi) \textit{ wf} \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi \textit{ wf} \text{ and } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi \textit{ wf}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} (\forall x_i .\, \varphi) \textit{ wf} \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i := a]} \varphi \textit{ wf} \text{ for all } a \in A$$

Semantics of T – III

$$\not\models^{\mathsf{T}}_{\mathfrak{M},\rho} \bot$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} P_i(t_1, \ldots, t_{r_i}) \quad \text{iff} \quad \left([\![t_1]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}, \ldots, [\![t_{r_i}]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}\right) \in [\![P_i]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} t_1 = t_2 \quad \text{iff} \quad [\![t_1]\!]^{\mathsf{T}}_{\mathfrak{M},\rho} = [\![t_2]\!]^{\mathsf{T}}_{\mathfrak{M},\rho}$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi \to \psi \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} (\varphi \to \psi) \text{ \textit{wf} and } \not\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi \text{ or } \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$$

$$\models^{\mathsf{T}}_{\mathfrak{M},\rho} \forall x_i.\, \varphi \quad \text{iff} \quad \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]} \varphi \text{ for all } a \in A$$

Semantics of $\mathcal{T}$ – IV

Semantics of T – IV

Well-formation of contexts:

1. $\epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
2. $\varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
3. $x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]}$ *wf* for all $a \in A$

Semantics of T – IV

Well-formation of contexts:

1. $\epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
2. $\varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
3. $x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]}$ *wf* for all $a \in A$

$\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf* defined in a similar way with

$1'.$ $\epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf*

Semantics of T *– IV*

Well-formation of contexts:

1. $\epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
2. $\varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf*
3. $x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho}$ *wf* iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]}$ *wf* for all $a \in A$

$\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf* defined in a similar way with

$1'$. $\epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} t$ *wf*

$\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ *wf* defined by $1'$, 2 and 3

Semantics of τ – V

Semantics of *T* – V

Consequence:

$1'.\quad \epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$2'.\quad \varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff

$(a)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \neg\varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ ***wf***

$(b)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$3.\quad x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\psi}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]} \psi$ for all $a \in A$

Semantics of T – V

Consequence:

$1'.\quad \epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$2'.\quad \varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff

$(a)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \neg\varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ *wf*

$(b)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$3.\quad x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\psi}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]} \psi$ for all $a \in A$

$\Gamma \models^{\mathsf{T}}_{\mathfrak{M}} \mathcal{X}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\emptyset} \mathcal{X}$

Semantics of T *– V*

Consequence:

$1'.\quad \epsilon \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff $\models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$2'.\quad \varphi, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ iff

$(a)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \neg\varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$ *wf*

$(b)\quad \models^{\mathsf{T}}_{\mathfrak{M},\rho} \varphi$ and $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho} \psi$

$3.\quad x_i, \Gamma \models^{\mathsf{T}}_{\mathfrak{M},\psi}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\rho[x_i:=a]} \psi$ for all $a \in A$

$\Gamma \models^{\mathsf{T}}_{\mathfrak{M}} \mathcal{X}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M},\emptyset} \mathcal{X}$

$\Gamma \models^{\mathsf{T}} \mathcal{X}$ iff $\Gamma \models^{\mathsf{T}}_{\mathfrak{M}} \mathcal{X}$ for all T-models $\mathfrak{M}$.

Semantics of D

Semantics of D

- Similar to T, but functions now may be partial:

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}} : A^n \not\to A$$

Semantics of D

- Similar to T, but functions now may be partial:

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}} : A^n \not\to A$$

- D_fs become ‘relevant’:

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}}(a_1, \ldots, a_n)\downarrow \text{ iff } (a_1, \ldots, a_n) \in [\![D_f]\!]^{\mathsf{D}}_{\mathfrak{M}}$$

Semantics of D

- Similar to T, but functions now may be partial:

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}} : A^n \not\to A$$

- D_fs become 'relevant':

$$[\![f]\!]^{\mathsf{D}}_{\mathfrak{M}}(a_1, \ldots, a_n) \downarrow \text{ iff } (a_1, \ldots, a_n) \in [\![D_f]\!]^{\mathsf{D}}_{\mathfrak{M}}$$

(modulo some technical problems. . .)

Equivalence of T *and* FOL

Equivalence of T and FOL

$$\cdot^{\circ} : \mathcal{T}_{\mathsf{T}} \quad \longrightarrow \quad \wp\left(\mathcal{L}_{\mathsf{FOL}} \times \mathcal{T}_{\mathsf{FOL}}\right)$$

Equivalence of T and FOL

$$
\begin{array}{rcl}
\cdot^{\circ} : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \wp\left(\mathcal{L}_{\mathsf{FOL}} \times \mathcal{T}_{\mathsf{FOL}}\right) \\
f_i(t_1, \ldots, t_{a_i}) & \mapsto & \left\{ \langle \bigwedge_{k=1}^{a_i} \psi_k, f_i(t'_1, \ldots, t'_{a_i}) \rangle \right\}
\end{array}
$$

Equivalence of T and FOL

$$
\begin{aligned}
\cdot^\circ : \mathcal{T}_{\mathsf{T}} \quad &\longrightarrow \quad \wp\left(\mathcal{L}_{\mathsf{FOL}} \times \mathcal{T}_{\mathsf{FOL}}\right) \\
f_i(t_1, \ldots, t_{a_i}) \quad &\mapsto \quad \left\{ \langle \bigwedge_{k=1}^{a_i} \psi_k, f_i(t'_1, \ldots, t'_{a_i}) \rangle \right\} \\
(\text{if } \varphi \text{ then } t_1 \text{ else } t_2) \quad &\mapsto \quad \{\langle \varphi^\circ \wedge \psi, t'_1 \rangle \mid \langle \psi, t'_1 \rangle \in {t_1}^\circ\} \cup \\
&\qquad \{\langle \neg\varphi^\circ \wedge \psi, t'_2 \rangle \mid \langle \psi, t'_2 \rangle \in {t_2}^\circ\}
\end{aligned}
$$

Equivalence of *T* and *FOL*

$$
\begin{array}{rcl}
\cdot^\circ : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \wp\left(\mathcal{L}_{\mathsf{FOL}} \times \mathcal{T}_{\mathsf{FOL}}\right) \\
f_i(t_1, \ldots, t_{a_i}) & \mapsto & \left\{ \langle \bigwedge_{k=1}^{a_i} \psi_k, f_i(t'_1, \ldots, t'_{a_i}) \rangle \right\} \\
(\text{if } \varphi \text{ then } t_1 \text{ else } t_2) & \mapsto & \{\langle \varphi^\circ \wedge \psi, t'_1 \rangle \mid \langle \psi, t'_1 \rangle \in {t_1}^\circ\} \cup \\
 & & \{\langle \neg\varphi^\circ \wedge \psi, t'_2 \rangle \mid \langle \psi, t'_2 \rangle \in {t_2}^\circ\}
\end{array}
$$

$$
\cdot^\circ : \mathcal{L}_{\mathsf{T}} \longrightarrow \mathcal{L}_{\mathsf{FOL}}
$$

Equivalence of *T* and *FOL*

$$
\begin{array}{rcl}
\cdot^\circ : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \wp\left(\mathcal{L}_{\mathsf{FOL}} \times \mathcal{T}_{\mathsf{FOL}}\right) \\
f_i(t_1, \ldots, t_{a_i}) & \mapsto & \left\{ \langle \bigwedge_{k=1}^{a_i} \psi_k, f_i(t'_1, \ldots, t'_{a_i}) \rangle \right\} \\
(\text{if } \varphi \text{ then } t_1 \text{ else } t_2) & \mapsto & \{\langle \varphi^\circ \wedge \psi, t'_1 \rangle \mid \langle \psi, t'_1 \rangle \in t_1{}^\circ\} \cup \\
 & & \quad \{\langle \neg\varphi^\circ \wedge \psi, t'_2 \rangle \mid \langle \psi, t'_2 \rangle \in t_2{}^\circ\}
\end{array}
$$

$$
\begin{array}{rcl}
\cdot^\circ : \mathcal{L}_{\mathsf{T}} & \longrightarrow & \mathcal{L}_{\mathsf{FOL}} \\
P_i(t_1, \ldots, t_{r_i}) & \mapsto & \bigwedge_{\langle \varphi_k, t'_k \rangle \in t_k{}^\circ} \left(\bigwedge_{k=1}^{r_i} \varphi_i \longrightarrow P_i(t'_1, \ldots, t'_{r_i}) \right)
\end{array}
$$

Completeness of T

Completeness of T

Let Γ be a context in T and $\varphi \in \mathcal{L}_{\mathsf{T}}$ such that $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*. Then $\Gamma \vdash^{\mathsf{T}} \varphi$ iff $\Gamma \models^{\mathsf{T}} \varphi$.

Completeness of T

Let Γ be a context in T and $\varphi \in \mathcal{L}_{\mathsf{T}}$ such that $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*.
Then $\Gamma \vdash^{\mathsf{T}} \varphi$ iff $\Gamma \models^{\mathsf{T}} \varphi$.

$\Gamma \vdash^{\mathsf{T}} \varphi$ iff (1) $\Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ}$ and (2) $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*.

Completeness of T

Let Γ be a context in T and $\varphi \in \mathcal{L}_{\mathsf{T}}$ such that $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*. Then $\Gamma \vdash^{\mathsf{T}} \varphi$ iff $\Gamma \models^{\mathsf{T}} \varphi$.

$\Gamma \vdash^{\mathsf{T}} \varphi$ iff (1) $\Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ}$ and (2) $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*.

- (1) is equivalent to $\Gamma^{\circ} \models^{\mathsf{FOL}} \varphi^{\circ}$.

Completeness of T

Let Γ be a context in T and $\varphi \in \mathcal{L}_{\mathsf{T}}$ such that $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*. Then $\Gamma \vdash^{\mathsf{T}} \varphi$ iff $\Gamma \models^{\mathsf{T}} \varphi$.

$\Gamma \vdash^{\mathsf{T}} \varphi$ iff (1) $\Gamma^{\circ} \vdash^{\mathsf{FOL}} \varphi^{\circ}$ and (2) $\Gamma \vdash^{\mathsf{T}} \varphi$ *wf*.

- (1) is equivalent to $\Gamma^{\circ} \models^{\mathsf{FOL}} \varphi^{\circ}$.
- (2) is equivalent to $\Gamma \models^{\mathsf{T}} \varphi$ *wf*.

Completeness of T

Let Γ be a context in T and $\varphi \in \mathcal{L}_\mathsf{T}$ such that $\Gamma \vdash^\mathsf{T} \varphi$ *wf*. Then $\Gamma \vdash^\mathsf{T} \varphi$ iff $\Gamma \models^\mathsf{T} \varphi$.

$\Gamma \vdash^\mathsf{T} \varphi$ iff (1) $\Gamma^\circ \vdash^\mathsf{FOL} \varphi^\circ$ and (2) $\Gamma \vdash^\mathsf{T} \varphi$ *wf*.

- (1) is equivalent to $\Gamma^\circ \models^\mathsf{FOL} \varphi^\circ$.
- (2) is equivalent to $\Gamma \models^\mathsf{T} \varphi$ *wf*.

The conjunction of these two holds iff $\Gamma \models^\mathsf{T} \varphi$.

Equivalence of T and D

Equivalence of T and D

$$\cdot^* : \mathcal{T}_{\mathsf{T}} \quad \longrightarrow \quad \mathcal{T}_{\mathsf{D}}$$

Equivalence of T and D

$$
\begin{array}{rcl}
\cdot^* : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \mathcal{T}_{\mathsf{D}} \\
f_i(t_1, \ldots, t_{a_i}) & \longmapsto & \text{if } D_{f_i}({t_1}^*, \ldots, {t_{a_i}}^*) \\
 & & \text{then } f_i({t_1}^*, \ldots, {t_{a_i}}^*) \text{ else } c_1
\end{array}
$$

Equivalence of T and D

$$
\begin{array}{rcl}
\cdot^* : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \mathcal{T}_{\mathsf{D}} \\
f_i(t_1, \ldots, t_{a_i}) & \mapsto & \text{if } D_{f_i}({t_1}^*, \ldots, {t_{a_i}}^*) \\
 & & \text{then } f_i({t_1}^*, \ldots, {t_{a_i}}^*) \text{ else } c_1 \\
\text{if } \vartheta \text{ then } t_1 \text{ else } t_2 & \mapsto & \text{if } \vartheta^* \text{ then } {t_1}^* \text{ else } {t_2}^*
\end{array}
$$

Equivalence of T and D

$$
\begin{aligned}
.^* : \mathcal{T}_{\mathsf{T}} &\longrightarrow \mathcal{T}_{\mathsf{D}} \\
f_i(t_1, \ldots, t_{a_i}) &\longmapsto \text{if } D_{f_i}({t_1}^*, \ldots, {t_{a_i}}^*) \\
&\qquad \text{then } f_i({t_1}^*, \ldots, {t_{a_i}}^*) \text{ else } c_1 \\
\text{if } \vartheta \text{ then } t_1 \text{ else } t_2 &\longmapsto \text{if } \vartheta^* \text{ then } {t_1}^* \text{ else } {t_2}^*
\end{aligned}
$$

$$
.^* : \mathcal{L}_{\mathsf{T}} \longrightarrow \mathcal{L}_{\mathsf{D}}
$$

Equivalence of T and D

$$
\begin{array}{rcl}
\cdot^* : \mathcal{T}_\mathsf{T} & \to & \mathcal{T}_\mathsf{D} \\
f_i(t_1, \ldots, t_{a_i}) & \mapsto & \text{if } D_{f_i}({t_1}^*, \ldots, {t_{a_i}}^*) \\
 & & \text{then } f_i({t_1}^*, \ldots, {t_{a_i}}^*) \text{ else } c_1 \\
\text{if } \vartheta \text{ then } t_1 \text{ else } t_2 & \mapsto & \text{if } \vartheta^* \text{ then } {t_1}^* \text{ else } {t_2}^*
\end{array}
$$

$$
\begin{array}{rcl}
\cdot^* : \mathcal{L}_\mathsf{T} & \to & \mathcal{L}_\mathsf{D} \\
\bot & \mapsto & \bot \\
P_i(t_1, \ldots, t_{r_i}) & \mapsto & P_i({t_1}^*, \ldots, {t_{r_i}}^*) \\
\varphi \to \psi & \mapsto & \varphi^* \to \psi^* \\
\forall x_i .\, \varphi & \mapsto & \forall x_i .\, \varphi^*
\end{array}
$$

The domain conditions

The domain conditions

$$
\begin{array}{rcl}
\mathcal{DC}^{\vdash}_{\Gamma} : \mathcal{T}_{\mathsf{T}} & \longrightarrow & \wp(\mathcal{J}_{\mathsf{T}}) \\
\mathcal{DC}^{\vdash}_{\Gamma}(f_i(t_1, \ldots, t_{a_i})) & = & \mathcal{DC}^{\vdash}_{\Gamma}(t_1) \cup \ldots \cup \mathcal{DC}^{\vdash}_{\Gamma}(t_{a_i}) \cup \\
 & & \cup \left\{ \Gamma \vdash^{\mathsf{T}} D_{f_i}(t_1, \ldots, t_{a_i}) \right\} \\
\mathcal{DC}^{\vdash}_{\Gamma}(\textsf{if } \vartheta \textsf{ then } t_1 \textsf{ else } t_2) & = & \mathcal{DC}^{\vdash}_{\Gamma}(\vartheta) \cup \mathcal{DC}^{\vdash}_{\Gamma,\vartheta}(t_1) \cup \mathcal{DC}^{\vdash}_{\Gamma,\neg\vartheta}(t_2)
\end{array}
$$

The domain conditions

$$
\begin{aligned}
\mathcal{DC}^{\vdash}_{\Gamma} : \mathcal{T}_{\mathsf{T}} &\longrightarrow \wp(\mathcal{J}_{\mathsf{T}}) \\
\mathcal{DC}^{\vdash}_{\Gamma}(f_i(t_1, \ldots, t_{a_i})) &= \mathcal{DC}^{\vdash}_{\Gamma}(t_1) \cup \ldots \cup \mathcal{DC}^{\vdash}_{\Gamma}(t_{a_i}) \cup \\
&\quad \cup \left\{\Gamma \vdash^{\mathsf{T}} D_{f_i}(t_1, \ldots, t_{a_i})\right\} \\
\mathcal{DC}^{\vdash}_{\Gamma}(\text{if } \vartheta \text{ then } t_1 \text{ else } t_2) &= \mathcal{DC}^{\vdash}_{\Gamma}(\vartheta) \cup \mathcal{DC}^{\vdash}_{\Gamma,\vartheta}(t_1) \cup \mathcal{DC}^{\vdash}_{\Gamma,\neg\vartheta}(t_2)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{DC}^{\vdash}_{\Gamma} : \mathcal{L}_{\mathsf{T}} &\longrightarrow \wp(\mathcal{J}_{\mathsf{T}}) \\
\mathcal{DC}^{\vdash}_{\Gamma}(P_i(t_1, \ldots, t_{r_i})) &= \mathcal{DC}^{\vdash}_{\Gamma}(t_1) \cup \ldots \cup \mathcal{DC}^{\vdash}_{\Gamma}(t_{r_i}) \\
\mathcal{DC}^{\vdash}_{\Gamma}(\forall x_i .\, \varphi) &= \mathcal{DC}^{\vdash}_{\Gamma,x_i}(\varphi)
\end{aligned}
$$

The domain conditions (cont.)

The domain conditions (cont.)

$$
\begin{aligned}
\mathcal{DC}^{\vdash}(\epsilon) &= \emptyset \\
\mathcal{DC}^{\vdash}(\Gamma, x_i) &= \mathcal{DC}^{\vdash}(\Gamma) \\
\mathcal{DC}^{\vdash}(\Gamma, \varphi) &= \mathcal{DC}^{\vdash}(\Gamma) \cup \mathcal{DC}^{\vdash}_{\Gamma}(\varphi)
\end{aligned}
$$

The domain conditions (cont.)

$$\begin{aligned} \mathcal{DC}^{\vdash}(\epsilon) &= \emptyset \\ \mathcal{DC}^{\vdash}(\Gamma, x_i) &= \mathcal{DC}^{\vdash}(\Gamma) \\ \mathcal{DC}^{\vdash}(\Gamma, \varphi) &= \mathcal{DC}^{\vdash}(\Gamma) \cup \mathcal{DC}^{\vdash}_{\Gamma}(\varphi) \end{aligned}$$

Theorem [Wiedijk & Zwanenburg 2003]:

$\Gamma \vdash^{\mathsf{D}} \varphi$ iff $\mathcal{DC}(\Gamma)$ and $\mathcal{DC}_{\Gamma}(\varphi)$ hold and $\Gamma \vdash^{\mathsf{T}} \varphi$.

Transformation of models

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{[\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*}\}$$

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{ [\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*} \}$$

$$[\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}_*}(e_1, \ldots, e_{a_i}) = \begin{cases} [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) & [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) \downarrow \\ [\![c_1]\!]^{\mathsf{D}}_{\mathfrak{M}} & \text{otherwise} \end{cases}$$

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{[\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*}\}$$

$$[\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}_*}(e_1, \ldots, e_{a_i}) = \begin{cases} [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) & [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) \downarrow \\ [\![c_1]\!]^{\mathsf{D}}_{\mathfrak{M}} & \text{otherwise} \end{cases}$$

For any ρ,

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{[\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*}\}$$

$$[\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}_*}(e_1, \ldots, e_{a_i}) = \begin{cases} [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) & [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) \downarrow \\ [\![c_1]\!]^{\mathsf{D}}_{\mathfrak{M}} & \text{otherwise} \end{cases}$$

For any ρ,

- $[\![t^*]\!]^{\mathsf{D}}_{\mathfrak{M},\rho} \simeq [\![t]\!]^{\mathsf{T}}_{\mathfrak{M}_*,\rho}$

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{[\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*}\}$$

$$[\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}_*}(e_1, \ldots, e_{a_i}) = \begin{cases} [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) & [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) \downarrow \\ [\![c_1]\!]^{\mathsf{D}}_{\mathfrak{M}} & \text{otherwise} \end{cases}$$

For any ρ,

- $[\![t^*]\!]^{\mathsf{D}}_{\mathfrak{M},\rho} \simeq [\![t]\!]^{\mathsf{T}}_{\mathfrak{M}_*,\rho}$
- $\models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi^*$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M}_*,\rho} \varphi$ *wf*

Transformation of models

Let $\mathfrak{M} = \langle A, F, P, C \rangle$ be a D-model. Then $\mathfrak{M}_*$ is the T-model defined by $\mathfrak{M}_* = \langle A, F_*, P, C \rangle$, where

$$F_* = \{[\![f_1]\!]^{\mathsf{T}}_{\mathfrak{M}_*}, \ldots, [\![f_n]\!]^{\mathsf{D}}_{\mathfrak{M}_*}\}$$

$$[\![f_i]\!]^{\mathsf{T}}_{\mathfrak{M}_*}(e_1, \ldots, e_{a_i}) = \begin{cases} [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) & [\![f_i]\!]^{\mathsf{D}}_{\mathfrak{M}}(e_1, \ldots, e_{a_i}) \downarrow \\ [\![c_1]\!]^{\mathsf{D}}_{\mathfrak{M}} & \text{otherwise} \end{cases}$$

For any ρ,

- $[\![t^*]\!]^{\mathsf{D}}_{\mathfrak{M},\rho} \simeq [\![t]\!]^{\mathsf{T}}_{\mathfrak{M}_*,\rho}$
- $\models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi^*$ *wf* iff $\models^{\mathsf{T}}_{\mathfrak{M}_*,\rho} \varphi$ *wf*
- $\models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi^*$ iff $\models^{\mathsf{T}}_{\mathfrak{M}_*,\rho} \varphi$

Future work

Future work

- find 'right' definition of $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$

Future work

- find 'right' definition of $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$
- define semantic domain conditions

Future work

- find ‘right’ definition of $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$
- define semantic domain conditions
- prove equivalence between T and D semantically

Future work

- find 'right' definition of $\Gamma \models^{\mathsf{D}}_{\mathfrak{M},\rho} \varphi$
- define semantic domain conditions
- prove equivalence between T and D semantically
- completeness of D