formalizing approximation fixpoint theory in coq

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types for proofs and programs
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in a nutshell

approximation fixpoint theory

- what
- why
- how

coq formalization

- guiding principles
- current status
- challenges

approximation fixpoint theory

what

a framework for studying fixpoints

- of operators over complete lattices
- of approximators to these operators

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unifying theory for many different constructions in (non-monotonic) logics

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how

heavy use of transfinite sequences and transfinite induction

the coq formalization

design decisions

- constructive (as far as possible)
- follow the mathematical development closely

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current status

- type of unbounded sets of ordinals
  - examples: ω, ω^ω
- type of complete lattices [very standard!]
  - example: powersets
  - constructions: billatice, dual
- basic definitions of aft (see abstract)

the coq formalization

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main challenges

adapt proofs relying on some classical decidability properties

thank you!