Coflow, Covering Vertices by Directed Circuits, and a Lower Bound on the Stability Number of a Graph

Kathie Cameron

(joint work with Jack Edmonds)

Let G be a digraph, and for each edge e of G, let $d(e)$ be a non-negative integer. The capacity, $d(C)$, of a dicircuit C means the sum of the $d(e)$'s of the edges in C. A version of the Coflow Theorem (1982) says:

Coflow Theorem. *The maximum size of a subset S of vertices of digraph G such that each dicircuit C of G contains at most $d(C)$ members of S equals the minimum of the sum of the capacities of any subset H of dicircuits of G plus the number of vertices of G which are not in a dicircuit of H.*

When we proved the Coflow Theorem, we hoped to prove the following conjecture made by Gallai in 1963:

Gallai's Conjecture. *For any digraph G such that each edge and each vertex is in a dicircuit, the maximum number of vertices in G no two of which are joined by an edge is at least as big as the minimum number of dicircuits which together cover all the vertices.*

However, we were missing the following:

Lemma. *For any digraph G such that each edge and each vertex is in a dicircuit, G contains a set F of edges such that $G - F$ is acyclic and every edge of G is in some dicircuit which contains exactly one edge of F.*

We recently learned that Knuth proved this lemma in 1974. The Coflow Theorem together with Knuth's Lemma provides a proof of Gallai's Conjecture different from that recently published by Bessy and Thomassé.