Partitions and bichromatic numbers of graphs

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Abstract

For a pair of integers $k, \ell \geq 0$, a graph $G = (V, E)$ is (k, ℓ)-colourable if V can be partitioned into $k + \ell$ (possibly empty) subsets $I_1, \ldots, I_k, C_1, \ldots, C_\ell$ such that each I_i induces an independent set and each C_j induces a clique in G. The (k, ℓ)-colourability, which generalizes both colouring and clique covering, best approximates the hereditary property of graphs. The bichromatic number $\chi^b(G)$ of G is the least integer r such that for all k, ℓ with $k+\ell = r$, G is (k, ℓ)-colourable. It is easy to see that $\chi^b(G)$ is bounded above by $\chi(G)+\theta(G)-1$ where $\chi(G)$ and $\theta(G)$ are respectively the chromatic number and the clique covering number of G. Here we characterize all graphs G for which the upper bound is attained, i.e., $\chi^b(G) = \chi(G) + \theta(G) - 1$. It turns out that these graphs are all cographs and they are critical in the sense that a cograph H is not (k, ℓ)-colourable if and only if H contains an induced subgraph G with $\chi(G) = k + 1$, $\chi(G) = \ell + 1$ and $\chi^b(G) = k + \ell + 1$.