

From Carsten's Proofs/Results to Hadwiger's Conjecture

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Abstract

Around 15 years ago, Carsten Thomassen proved that there are only finitely many 6-color-critical graphs on a fixed surface. The result itself is very deep, but the methods are also useful.

In this talk, we shall show how the CT's methods can be used to prove the following theorems concerning Hadwiger's conjecture (which says that every graph with no K_t -minor is $(t - 1)$ -colorable).

1. Every minimal counterexample to Hadwiger's conjecture has at most $f(t)$ vertices for some function f of t .
2. There is a polynomial time (actually, we believe an $O(n^2)$ time !) algorithm to decide Hadwiger's conjecture for fixed t .

In addition, we shall discuss the following topics:

1. Does every t -colorable graph with no K_t -minor have exponentially many colorings?
2. Additive approximation algorithm for list-coloring graphs with no K_t -minor.
3. t -colorability of graphs with no K_{t+1} -minor.

All of these topics are motivated by CT's work on coloring graphs in a fixed surface.