

# A negative solution of the edge-coloring conjecture of Grünbaum

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## **Abstract**

An embedding of a graph in a surface is called polyhedral if its dual has no multiple edges and loops. A conjecture of Grünbaum, presented in 1968, states that each 3-regular graph with a polyhedral embedding in an orientable surface has a 3-edge-coloring. This statement holds true for the sphere, because by Tait, the Four Color theorem is equivalent with the statement that each 2-edge-connected 3-regular planar graph has a 3-edge-coloring. Thus the Grünbaum's conjecture aims to generalize the Four Color Theorem for any orientable surface. We present a negative solution of this conjecture, showing that for each orientable surface of genus at least 5, there exists a 3-regular non 3-edge-colorable graph with a polyhedral embedding in it.