Generalized domination in special graph classes

Jan Kratochvíl
Charles University
Czech Republic

Joint work with
Petr Golovach

Abstract

We investigate the interplay of polynomial solvability and warranty of unique solution of the problem under consideration, in the setting of generalized domination.

Given sets σ, ρ of nonnegative integers (as parameters of the problem), a set S of vertices of a graph G is called (σ, ρ)*-dominating* if the number of S-neighbors of any vertex of S (of $V \setminus S$) is an element of σ (ρ, respectively). This notion was introduced by Telle and has been investigated by Telle, Proskurowski, Heggernes, Miller, etc. In particular, for any pair of finite nonempty sets σ, ρ (such that $0 \notin \rho$), already deciding the existence of a (σ, ρ)-dominating set in an input graph is NP-complete. Polynomial/NP-completeness dichotomy results for restricting the input graphs to be chordal (or k-degenerate) were obtained by Golovach and Kratochvil. They relate to the concept of ambivalence in the following sense.

Given a graph class M, the pair (σ, ρ) is called *ambivalent for* M if there exists a graph $G \in M$ with at least two different (σ, ρ)-dominating sets; otherwise it is *non-ambivalent for* M. For chordal graphs, the existence of a (σ, ρ)-dominating set can be decided in polynomial time when the pair (σ, ρ) is non-ambivalent for chordal graphs, and the problem is NP-complete otherwise. Similarly for k-degenerate graphs (for any $k \geq 2$).

The last part of the talk will deal with planar graphs, where we are not able to fully characterize the computational complexity, nor the connection to ambivalence. We believe that this leads to interesting open problems.