

# Injective homomorphisms of directed graphs

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## Abstract

A homomorphism from a digraph  $D$  to a digraph  $H$  is called *injective* if it is injective on the in-neighbourhood of each vertex. Complexity results for injective homomorphisms of irreflexive digraphs  $D$  are considered in the case when the target digraph  $H$  is reflexive, and in the case where the target graph  $H$  is irreflexive. A dichotomy theorem is obtained in the case where  $H$  is reflexive, whereas a such a theorem in the case where  $H$  is irreflexive would imply one for all digraph homomorphism problems. The complexity of the related *injective oriented chromatic number* problems (the minimum  $n$  for which a digraph  $D$  admits an injective homomorphism to a digraph on  $n$  vertices – defined together with A. Raspaud) is also discussed.