

# Independent Sets in 4-Regular Hamiltonian Graphs

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## Abstract

In the early 90s, Du, Hsu and Hwang raised the question whether 4-regular hamiltonian graphs whose inner cycles are triangles ("cycle-plus-triangles graphs") contain independent sets of size at least  $n/3$ , where  $n$  is the order of the graph. A well-known result, conjectured by Erdős and proved by Fleischner and Stiebitz, provides an affirmative answer in the stronger sense that cycle-plus-triangles graphs are 3-colorable. This result cannot be generalized to "smooth" 4-regular hamiltonian graphs (i.e. whose inner cycles - like triangles - are non-selfintersecting): it has been shown that for such graphs 3-colorability is NP-complete. This left open the extension to smooth 4-regular hamiltonian graphs of the original question of Du, Hsu and Hwang concerning independent sets. We show that for these graphs both the Maximum Independent Set Problem (MIS) and the Large Independent Set Problem (existence of an independent set of size at least  $n/3$ ) are NP-complete. As an auxiliary result we prove that MIS is NP-complete also for 3-regular hamiltonian graphs, indeed even for planar hamiltonian 3-regular graphs.