

Distance-Two Colouring of Graphs

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A *distance-two colouring* of a graph G is a colouring of the vertices of G in which vertices at distance one or two must get different colours. This is obviously the same as a normal (proper) vertex-colouring of the *square* G^2 of G , where G^2 is the graph with the same vertex set as G and with an edge between any two different vertices that have distance at most two in G . Finding the chromatic number of squares of graphs has been an area of intensive research, in particular for planar graphs.

Wegner conjectured in 1977 that the square of a planar graph has chromatic number at most $\frac{3}{2}\Delta(G) + 1$ for $\Delta(G) \geq 8$, a bound that would be best possible. We show it is at most $(\frac{3}{2} + o(1))\Delta(G)$, and indeed this is true for the list chromatic number and for more general classes of graphs.

In 1984, Borodin formulated a similar conjecture on so-called *cyclic colourings* of plane graphs, where vertices incident with the same face need to get different colours. In order to obtain similar asymptotic results for the cyclic chromatic number, we generalise the concept of distance-two colouring.

More specifically, we study the case that we are given a graph G and two sets $A, B \subseteq V(G)$ (not necessarily disjoint). And the requirement is to colour the vertices of B so that (i) adjacent vertices get different colours, and (ii) vertices with a common neighbour from A get different colours. For planar graph we can give asymptotically best possible upper bounds on the number of colours required for such colourings (in terms of the natural degree condition).

This talk is based on recent research done jointly with FRÉDÉRIC HAVET (INRIA, Sophia-Antipolis), COLIN MCDIARMID (University of Oxford) and BRUCE REED (McGill University, Montreal and INRIA, Sophia-Antipolis); and with OMID AMINI (Max-Planck-Institut für Informatik, Saarbrücken) and LOUIS ESPERET (LaBRI, Bordeaux).