

Grötzsch's 3-Colour Theorem in Terms of Integer Flows

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Joint work with
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Abstract

Our thread rises from two strands. The first is Carsten Thomassen's 2003 proof of Grötzsch's Theorem (1958). What is special about this proof is that, in place of an appeal to the Euler Polyhedron Formula, Carsten uses a set of colour restrictions on the boundary of the yet uncoloured submap. The proof shows how vertices along the boundary are coloured or the uncoloured submap severed while staying within the set of restrictions. Bruce Richter and I – coauthors in this research – tried to understand Thomassen's technique by constructing a proof using a little different conditions on the boundary. We were joined in this effort by Cândida Nunes da Silva: her fundamental contributions got our proof off the ground.

The second strand is the Steinberg-Younger 1989 proof of Grötzsch's Theorem in dual form, i. e., in terms of 3-flows. This proof, whose main focus was upon an extension to the projective plane, centrally appeals to the Euler Polyhedron Formula. Can this, for the planar case, be replaced by an adaptation of the Thomassen boundary technique? What conditions are maintained and how is the boundary advanced, in this integer flow context?

This talk describes the proof.