

Lower and upper bounds for chromatic number and some open problems

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For any positive integer k and orientation D of a graph G we denote by $\Delta_k(D)$ the largest value t for which there exists a directed path $P = v_1, v_2, \dots, v_k$ such that $d^+(v_k) = t$, where $d^+(v_k)$ stands for the out-degree of v_k . We first obtain an upper bound for the chromatic number of G in terms of $\Delta_k(D)$. Using this bound we present another upper bound in terms of a new parameter $\Delta_k^{\leftarrow}(G)$ involving the maximum degrees in G . We compare our bound with the coloring number bound and discuss the algorithmic aspects of $\Delta_k(D)$.

The next set of upper bounds are in terms of girth and the booksize of graphs. For any two integers $0 \leq t < k$ by the booksize $b_{t,k}(G)$ of a graph G we mean the maximum number of k -cycles say C_1, \dots, C_m such that for some path P of length t , $V(C_i) \cap V(C_j) = V(P)$ for any $i \neq j$. Using this concept we improve the best known bound in terms of girth for the chromatic number of graphs when girth is an even integer. We generalize the results for even-girth of graphs.

Finally we obtain some lower bounds in terms of maximum or average degree of graphs. We show that for any tree T and integer t the chromatic number of any $(T, K_{2,t})$ -free graph is lower bounded by a fraction of average degree. A lower bound is also given in terms of the maximum even-hole of graphs.

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