

The Potential of Greed for Independence

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The well-known lower bound on the independence number of a graph due to Caro (New Results on the Independence Number, Technical Report, Tel-Aviv University, 1979) and Wei (A Lower Bound on the Stability Number of a Simple Graph, Technical memorandum, TM 81 - 11217 - 9, Bell laboratories, 1981) can be established as a performance guarantee of two natural and simple greedy algorithms or of a simple randomized algorithm.

We study possible generalizations and improvements of these approaches using vertex weights and discuss conditions on so-called potential functions $p_G : V(G) \rightarrow \mathbb{N}_0$ defined on the vertex set of a graph G for which suitably modified versions of the greedy algorithms applied to G yield independent sets I with $|I| \geq \sum_{u \in V(G)} 1/(p_G(u) + 1)$.

We provide examples of such potentials, which lead to bounds improving the bound due to Caro and Wei. Furthermore, suitably adapting the randomized algorithm we give a short proof of Thiele's lower bound on the independence number of a hypergraph (A lower bound on the independence number of arbitrary hypergraphs, *J. Graph Theory* **30** (1999), 213-221).

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