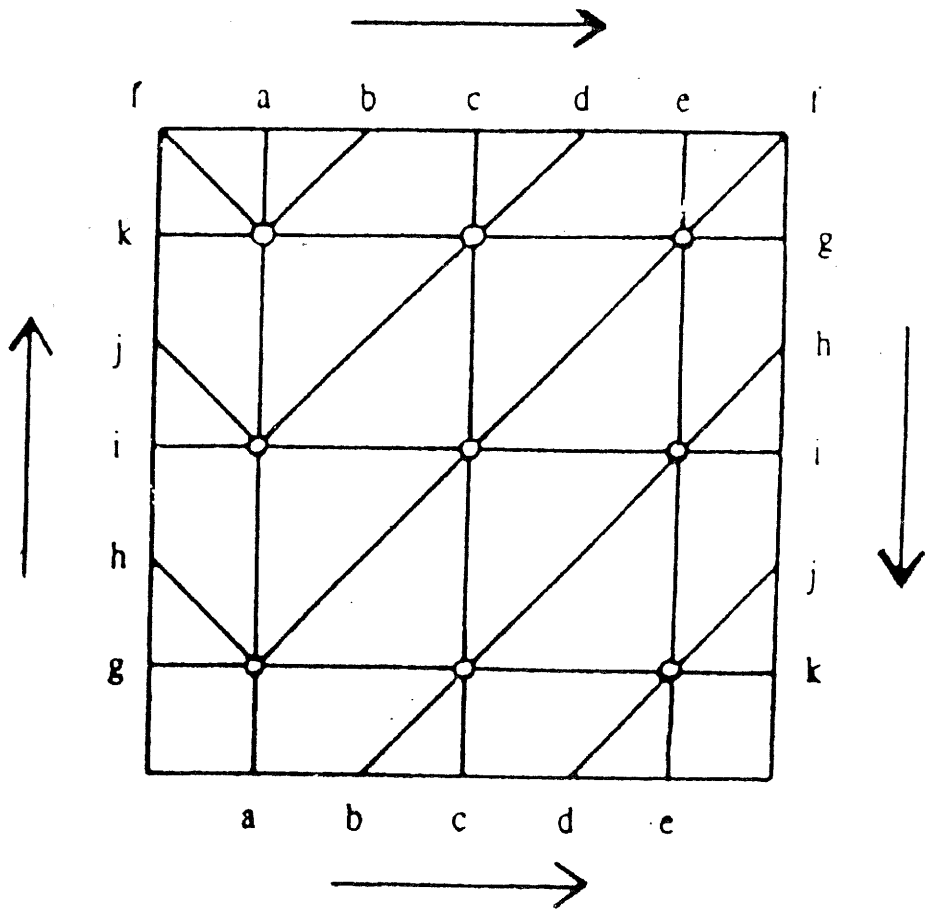
University of Colorado at Colorado Springs
and
Center for Excellence in Mathematical Education

GEOMBINATORICS



Volume V October 1995 Issue 2

GRAPH COLORING PROBLEMS

by Tommy R. Jensen and Bjarne Toft
John Wiley & Sons, New York, 1995;
xix+295 pp., ISBN 0-471-02865-7

A Review by Alexander Soifer

Problems and results are connected in an eternal chain of creation, just like chickens and eggs. Take away problems – and there will be no results, take away results – and there will be no problems.

A new book, *Graph Coloring Problems,* by the two Danish mathematicians Tommy R. Jensen and Bjarne Toft belongs to the genre of problem books. It is appropriately dedicated to Paul Erdős.

Mathematicians of the last few generations have been fortunate to live in the time of Paul Erdős, the greatest problem creator. Not only did his legendary travel and talks provide numerous mathematicians in many fields of mathematics with an abundance of open problems – Paul has also created a very special style of presenting open problems. In all of his "problem" talks and papers Erdős conveys the origin of his problems, history, partial results and some bibliography. As a result, the audience and the readers are ready and eager to sail off into their search for solutions and consequently new problems.

The first problem books of "Erdősian kind" appeared in early eighties and were dedicated to problems of number theory. *Old and New Problems and Results in Combinatorial Geometry* [EG] was written by Paul Erdős and Ronald L. Graham and published by University of Geneva (the authors are preparing a new publication which would be more readily available). *Unsolved Problems in Number Theory* [G] was written by Richard K. Guy.

Connoisseurs of geometry had to wait for another decade to see problem books in their field published. But then they got two books

almost at the same time: *Unsolved Problems in Geometry* [CFG] by Hallard T. Croft, Kenneth Falconer and Richard K. Guy; and *Old and New Unsolved Problems in Plane Geometry and Number Theory* [KW] by Victor Klee and Stan Wagon.

Of course, there is also *"The Book"*, or *Research Problems in Discrete Geometry*. (It is probably called "The Book" because it has never been published as a book). This is a Xerox copied collection of wonderful problems that originally, in 1977, contained 14 problems proposed by Leo Moser, then expanded a few times by William O. J. Moser, and the 1992 version contains 113 problems of various authors assembled by Willie Moser and Janos Pach. The authors have planned for many years to make a book out of "The Book" and have it published by The American Mathematical Society. I hope they will!

The book reviewed here is not the first work Bjarne Toft has written on the subject. He contributed a chapter *75 Graph-Colouring Problems* to the 1990 book [NW]. While the chapter was a valuable contribution at the time, it now appears as a rehearsal for a larger and better work, the current book. The main difference is not just an increase from 75 problems to over 200. The problems in the 1990 are often presented in overly concise form, with very limited history and motivation. As a result I am not sure that a non-specialist could appreciate many of the problems and become interested enough in a problem to dedicate months or years of his or her life to working on it.

The new book by Bjarne Toft and his former student Tommy Jensen is an entirely different matter. Not only its 300+ pages and 17 chapters (each with its own bibliography) present a wealth of material. The book works precisely as a problem book should work. It excites and motivates, and when you are hooked, it gives you just the right survey of known results and sources to start your work on your favorite problems from this book.

Of course, the book presumes some prior acquaintance with graph theory. For those young readers who need their first introduction, I recommend a beautifully written *Graph Theory, 1736-1936* [BLW] by N. L. Biggs, E. K. Lloyd and R. J. Wilson. Following an introduction, it

is a fair game for all: Toft-Jensen book is a level playing field for a mature professional and immature youngster. Everyone is equal at the start because so many of the problems in the book require little or no knowledge to understand them, and a lot of creativity to conquer.

A wonderful book! My gratitude and congratulations to the authors.

Bibliography

[BLW] Biggs, N. L., Lloyd, E. K. and Wilson, R. J. *Graph Theory, 1736-1936*, Oxford: Clarendon Press, 1976.

[CFG] Croft, H. T., Falconer, K. and Guy, R. K. *Unsolved Problems in Geometry*, New York: Springer, 1991.

[EG] Erdős, P. and Graham, R. L. *Old and New Problems and Results in Combinatorial Geometry*, Geneva: L'Enseignement Mathematique, 1980.

[G] Guy, R. K. *Unsolved Problems in Number Theory*, New York: Springer, 1980.

[KW] Klee, V. and Wagon, S. *Old and New Unsolved Problems in Plane Geometry and Number Theory*, Washington, DC: Math. Assoc. Amer., 1991.

[MP] Moser, W. O. J. and Pach, J. *"The Book"* (*Research Problems in Discrete Geometry*), McGill University copy machine, 1992.

[NW] Nelson, R. and R. J. Wilson (Ed's). *Graph Colourings*, Essex: Longmont, 1990.

ATTENTION AUTHORS, PUBLISHERS AND READERS:

As you can see, with this issue we have started a new department:

Book Reviews

You are invited to submit for our review your new or recent problem books, books on combinatorial or discrete geometry, geometric combinatorics and related areas.
