Assignment No. 1
Parallel Computing, DM8XX (Fall 2008)

Department of Mathematics and Computer Science
University of Southern Denmark
Daniel Merkle

Due on: 18. September, 12:00 p.m. (Department secretaries office (Lone Seidler Petterson) or my office).

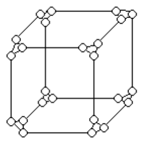
Note, that the assignments must be passed in order to take the oral exam. You must earn i.) 50% of all possible points of all assignments and ii.) 30% of each assignment to pass the assignments. Exercises or parts of exercises marked with * are voluntary exercises.

For preparing solutions for the following exercises you need to read the complete chapter 2 of the course book.

Exercise 1

Static Network Topology (5+5+5+5=20 points)

We discussed the static network topology d-dimensional hypercube. A disadvantage of a hypercube-based architecture is that the degree grows with growing values for d. A modification of the hypercube is the so called cube-connected cycle graph (CCC-graph), that doesn't have that property.



Cube-connected cycle graph for $d = 3$

For a CCC-graph each node of a hypercube is replaced by a ring of d nodes. The set of nodes is defined by $V_{CCC} = V_H \times \{0, \ldots, d-1\}$. In a CCC-graph there is a edge between nodes (v, i) and (v', i') if

- $i = i'$ and v and v' differ only in the $i-$th bit, or
- $|i - i'| = 1$ and $v = v'$, or
- $i - i' = d$ and $v = v'$.

a) Determine the number of shortest paths between two nodes v_1 and v_2 in a standard hypercube. Hint: use the binary node labeling.

b) Determine the number of nodes and edges in a CCC-graph and of a hypercube with dimensionality d.

c) Determine the diameter of a CCC-graph for $d = 3$.

d) Determine the diameter of a CCC-graph for $d \geq 4$. Explain how you derived the formula and give an example for the case of $d = 4$. (A proof would be very complicated and is not necessary).

e) * Explain an optimal routing algorithm in a CCC-graph. (The solution for this question is quite complicated, we won't discuss the solution for this part in detail.)

Exercise 2

Routing (10 points)

Consider the routing of messages in a parallel computer that uses store-and-forward routing. In such a network, the cost of sending a single message of size m from P_{source} to $P_{\text{destination}}$ via a path of length d is $t_s + t_w \times d \times m$. An alternate way of sending a message of size m is as follows. The user breaks the message into k parts each of size m/k, and then sends these k distinct messages one by one from P_{source} to $P_{\text{destination}}$. For this new method, derive the expression for the time to transfer a message of size m to a node d hops away under the following two cases:

a) Assume that another message can be sent from P_{source} as soon as the previous message has reached the next node in the path.

b) Assume that another message can be sent from P_{source} only after the previous message has reached $P_{\text{destination}}$.

For each case, comment on the value of this expression as the value of k varies between 1 and m. Also, what is the optimal value of k if t_s is very large, or if $t_s = 0$?

Exercise 3

Mesh of Trees (5+5+5=15 points)

A mesh of trees is a network that imposes a tree interconnection on a grid of processing nodes. A $\sqrt{p} \times \sqrt{p}$ mesh of trees is constructed as follows. Starting with a $\sqrt{p} \times \sqrt{p}$ grid, a complete binary tree is imposed on each row of the grid. Then a complete binary tree is imposed on each column of the grid. Figure 1 illustrates the construction of a 4×4 mesh of trees. Assume that the nodes at intermediate levels are switching nodes. Determine the bisection width, diameter, and total number of switching nodes in a mesh of trees.

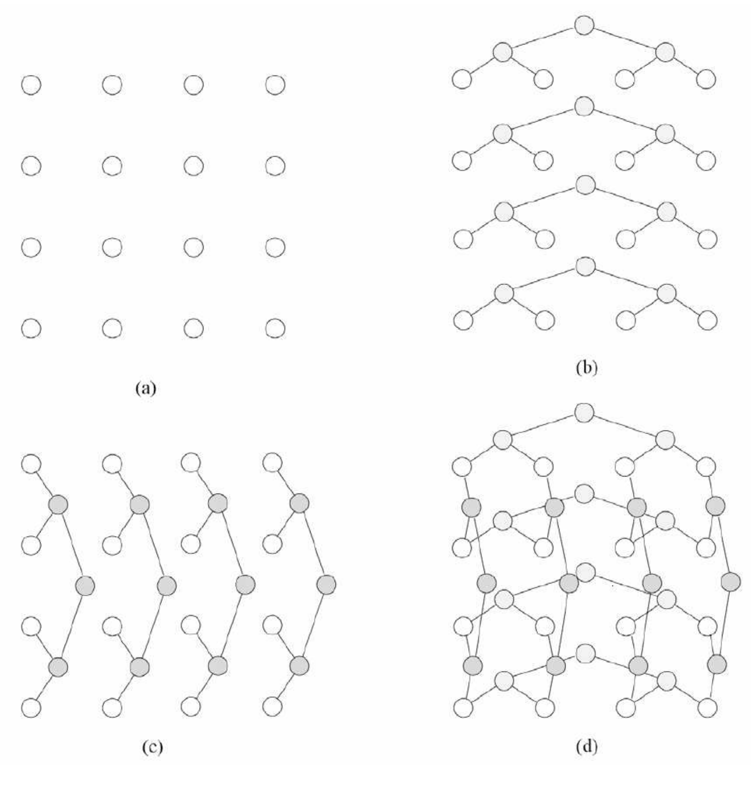


Figure 1: Mesh of Trees (Exercise 3)

Exercise 4

Embedding (5+5+5=15 points)

Let $G = (V_G, E_G)$ and $H = (V_H, E_H)$ be two graphs. An embedding of G into H is a pair of mappings (f_V, f_E), where $f_V : V_G \rightarrow V_H$ is a mapping of node sets and $f_E : E_G \rightarrow E_H^*$ maps edges in G to paths in H. For an embedding it holds that for every edge $e = (u, v) \in E_G$ the path $f_E(e)$ is a path from $f_V(u)$ to $f_V(v)$ in H.

a) Formalize congestion and dilation using the definition above.

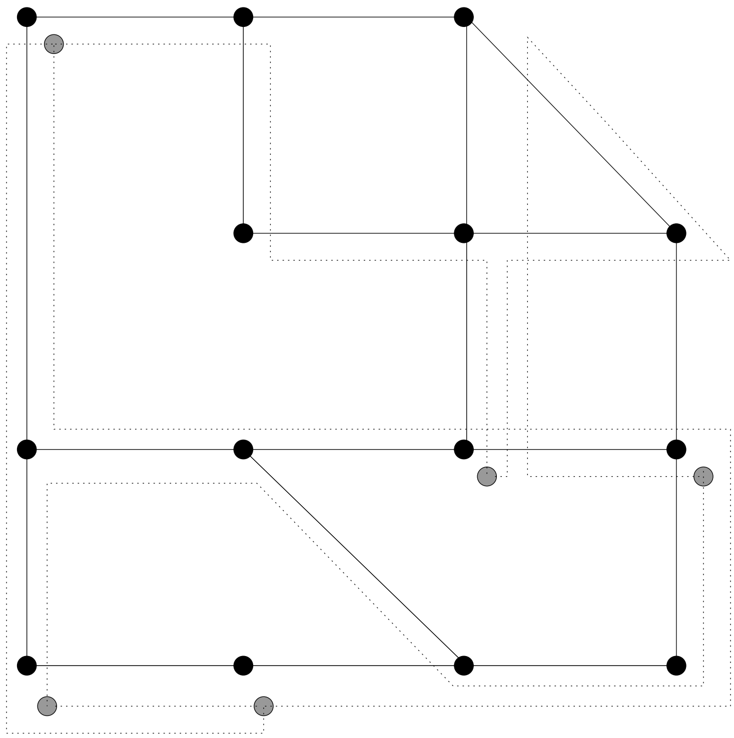


Figure 2: Embedding (Exercise 4)

b) Determine congestion and dilation of the embedding of graph G (dotted edges, gray nodes) into graph H (solid edges, black nodes) in Figure 2.

c) Proof the following: There is no injective embedding with dilation 1 of a complete binary tree with depth $d > 2$ (and having therefore $2^d - 1$ nodes) into an hypercube of dimension d.