Institut for Matematik og Datalogi
Syddansk Universitet

December 6, 2006
JBJ

DM19 – Fall06 – Weekly note 13

Stuff covered December 6, 2006

We finished Chapter 10 on heuristics in DM19 notes. Note that the proof of Theorem 10.1 is not pensum and also the Kerninghan-Lin heuristic for TSP is not pensum.

Exercises December 12, 2006

- Describe construction heuristics for the independent set problem.
- In the DM19 notes on Branch and Bound there is a TSP instance for the island of Bornholm. Try various of the TSP construction heuristics on this instance including the 2-approximation via a MST.
- Discuss the proof of Lemma 10.2 page 334 in notes for DM19 II
- Prove Theorem 10.3 on page 335.
- Let $\mathcal{A}$ be the usual descent algorithm for graph partitioning (swap an arbitrary pair which results in a decrease in the number of edges with one end in V_1 and the other in V_2). Vi assume that all edge weights are 1 and that our goal is to minimize the number of edges with one end in V_1 and the other in V_2. Prove that $\mathcal{A}$ will end up in a local minima after a polynomial number of swaps.
- Let $G = (V, E)$ be the following graph on 8 vertices: $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $E = \{ij : i \in \{1, 2, 3, 4\}, j \in \{5, 6, 7, 8\}\}$. Perform one iteration of the Kernighan -Lin algorithm on G starting from the partition $V_1 = \{1, 2, 3, 4\}$ and $V_2 = \{5, 6, 7, 8\}$.
- The **Linear ordering problem (LOP)** is the following given a directed graph $D = (V = \{1, 2, \ldots, n\}, A)$ and a non-negative weight function ω on A. Find a permutation π of V which maximizes

$$lop(D, \omega) = \sum_{\{ij \in A | \pi(i) < \pi(j)\}} \omega(ij),$$

  that is, the sum over the arcs the go forward in the ordering.

  1. Show how to solve LOP optimally when D is acyclic.
  2. Give a 2-approximation algorithm for the general case. Hint: consider any permutation and derive a good permutation from that in a simple way.

3. Describe 1-opt and 2-opt heuristics for LOP.

- The LOP is closely related to the **Feedback arc set problem (FAS)** where the goal is to minimize the weight of those arcs that go backwards in the ordering (instead of forwards). I.e. we wish to minimize

$$fas(D,\omega) = \sum_{\{ij \in A | \pi(i) > \pi(j)\}} \omega(ij)$$

  1. What is the precise relation between $fas(D,\omega)$ and $lop(D,\omega)$? (what about directed 2-cycles?).
  2. Can you get a good approximation algorithm for FAS via the 2-approximation algorithm for LOP? Hint: what is the only guaranteed lower bound for $fas(D,\omega)$?

- Describe a genetic algorithm for the MAX-SAT problem.

Lecture December 13, 2006

We will cover the notes on Branch and Bound and start with online algorithms from DM19 notes.

Test exam

The test exam will be Thursday December 21, 12.15-14 in U47. Note that this replaces the scheduled extra excercises that day (at the same time).

Exam questions

1. Information-theoretic lower bounds (lower bounds proven by counting leaves in decision trees), especially the average case bounds for sorting by comparisons.
2. Adversary arguments – technique, examples.
3. Median problem – algorithm and lower bound.
4. Fibonacci heaps.
5. String matching.
6. Huffman coding.
7. Proof that SATISFIABILITY is NP-complete.
8. NP-completeness proofs – examples.

9. Approximation algorithms for NP-complete problems.
10. Randomized algorithms.
11. Branch and bound.
12. Heuristics.
13. On-line algorithms.

The form of the exam

The exam will be January 24-26. These are the official dates but we will try to use only two days until we see the demand for one more day. The sign-up sheets for the order in which students will be examined are already in the secretaries' office. Sign up well before the exam, preferably before christmas.

The sign-up list cannot be used to exactly calculate an exam time since some students may not show up. Since, as for all classes, the external examiner is only paid for students who are examined, not for sitting and waiting, if a student is not there, the next student on the list who is present will be taken. When there are no more students ready to be taken, the external examiner may leave, so show up plenty early to make sure you are examined. Two hours before your expected exam time is probably safe enough.

You will draw a topic from the list of topics listed above. You will have 30 minutes to prepare your presentation. During this time you may use the book and your notes. during the exam you may also make short notes (at most 1 page) that will help you to organize your presentation, but that will have no other technical content. At any time the censor and I may ask you to stop using the notes if we feel the use is excessive.

The exam will take about 30 minutes per person. Prepare you presentation so that it takes no more than 20 minutes. Make sure you cover the most important ideas from your topic, though this may mean that you need to skip some details. Your presentation may be interupted with questions or cut short to go on to other topics. Towards the end of the 30 minute period, you will typically also be asked short questions not related to the material you talked about.

You may do your presentation in either Danish or English.

Pensum

The pensum listed on the faculty's pensum pages is not correct: All weekly notes and exercises on these is still part of the pensum.