Institut for Matematik & Datalogi
Syddansk Universitet

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JBJ

Combinatorial Optimization II (DM209)) — Ugeseddel 6

Stuff covered in Weeks 10 and 11

- The constrained minimum spanning tree problem: based on Ravi and Goemans [paper and Khuller Section 40.
- The steiner tree problem, Khuller section 42.
- The complexity of multicut problems. Based on handout of the paper Bang-Jensen and Yeo, the complexity of multicut and mixed multicut problems in digraphs, manuscript 2013 (submitted).

1 Final pensum

- Alexander Schrijver, A course in Combinatorial Optimization, notes from CWI Amsterdam 2010 pages 132-135,139-146, 148-158, 165-168.
- C. H. Papadimitriou and K. Steiglitz, Combinatorial Optimization, Prentice Hall 1982 pages 307-318, 414-419.
- J. Bang-Jensen and G. Gutin, Digraphs: Theory, Algorithms and Applications, Springer Verlag 2001 (PDF version available on course page): pages 35-38, 389, 404-409,476-482, 484-487.
- R. Niedermeier, Invitation to fixed parameter algorithms, Oxford University Press, 2006, 150-163.
- S. Khuller, Design and analysis of algorithms: course notes University of Maryland, pages 109-111, 115-119.
- S. Fortune, J. Hopcroft and J. Wyllie, The directed subgraph homeomorphism problem, Theoretical computer science **10** (1980) 111–121 (11 pages)
- Notes from Lecture 23 in 18.997 Topics in Combinatorial Optimizations (Lecturer Michel Goemans, Scribe Dan Stratila) on the Okamura-Seymour theorem. Including the Wagner-Weihe algorithm.
- J. Bang-Jensen and A. Yeo, The minimum spanning strong subdigraph problem is fixed parameter tractable Discrete Applied Mathematics, Volume 156 (2008) 2924-2929 (6 pages)

- J. Bang-Jensen and A. Yeo, Arc-disjoint spanning sub(di)graphs in digraphs, mansucript (available from the course page).
- J. Bang-Jensen and A. Yeo, The complexity of multicut and mixed multicut problems in (di)graphs, manuscript submitted to TCS.
- J. Bang-Jensen and S. Bessy, (Arc)-disjoint flows in networks, manuscript. Only Section 2 is pensum.
- J. Bang-Jensen, M. Kriesell, A. Maddaloni, S. Simonsen, vertex-disjoint directed and undirected cycles in general digraphs, submitted to J. Combinatorial Theory Ser. B. Only Section 4 is pensum.
- Chapter 4 of the book, Algorithmic Graph Theory and perfect graphs by M.C. Golumbic, Academic Press 1980.
- S. Thomassé, A quadratic kernel for feedback vertex set, SODA 2009 pages 115-119.
- Notes by Csaba Biró on Tree-width and grids (5 pages), taken from the home page of Robin Thomas (URL: http://people.math.gatech.edu/ thomas/TEACH/8863in05/notes0228.pdf)
- R. Ravi and M. Goemans, The Constrained minimum spanning tree problem, lecture Notes in Computer Science 1097 (1996) 66-75.
- All weekly notes including problems posed here and information on these.

2 Exam info

The exam will be roughly 30 minutes per person with 30 minutes preparation.

The exam starts at 8.45 on April 5 in U49D and IMADAs seminarroom. The first student (Mette) will draw an exam question at 8.15 and the exam will start at 8.45, where Uffe draws his question.

You will have about 15-20 minutes to cover the question you have drawn and then the external examinator and I will ask questions in other part of the pensum for about 10 minutes.

3 Exam questions

The sub-items below are suggestions/examples of what one could address at the exam and they are NOT an exhaustive list!

1. The k-path problem for directed graphs
   - Proof of NP-completeness of the 2-path-problem for directed graphs
   - Polynomial algorithm for the k-path problem in acyclic digraphs.
   - The complexity of the directed subgraph homeomorphism problem (Fortune, Hopcroft and Wyllie).
2. Multicommodity flows
   - Edge-disjoint paths as a special case.
   - Cut-condition, Euler condition.
   - Hu's 2-commodity flow theorem.
3. Edge-disjoint paths in planar graphs, the Okamura-Seymour Theorem
   - Cut-condition, Euler condition.
   - Use of planarity in the proof of the Okamura-Seymour Theorem.
   - Solving the edge-disjoint k-path problem in polynomial time for every fixed k (via the Robertson-Seymour Theorem and line graphs).
4. Approximation algorithms
   - Using Lagrange relaxation in approximation algorithms (constrained minimum spanning tree problem).
   - Finding small certificates for k-strong connectivity in digraphs (BJG chapter 7).

- Approximation algorithms for TSP and Steiner tree.

5. NP-completeness proofs
   - Steiner tree
   - 2-path problem for digraphs
   - Finding (s,t)-paths avoiding at least one vertex from prescribed sets of vertices and applications of this (Bang-Jensen, Yeo paper).
   - Disjoint directed and undirected cycles in a digraph (Bang-Jensen, Kriesell, Maddaloni, Simonsen).
   - Spanning connected eulerian subdigraph (Bang-Jensen and Bessy paper).
   - The complexity of multicut problems in (di)graphs (Bang-Jensen and Yeo)
6. Finding a subdivisions of a fixed (di)graph H in a (di)graph.
   - Relation to the k-path problem in (di)graphs
   - Polynomial algorithm for the problem for undirected graphs.
   - Complexity for digraphs in the general case and for acyclic digraphs.
7. Chordal graphs and tree-width
   - Recognizing chordal graphs (LexBFS)
   - Tree-representations of chordal graphs and how to obtain these.
   - Solving the chromatic number, maximum clique, minimum vertex cover and the maximum independent set problem for chordal graphs.
   - Tree-width of chordal graphs.
8. Tree width
   - Cops and robber game and relation to tree-width of a graph.
   - Heuristic algorithms for finding a good tree-decomposition (relation to chordal graphs).
   - Dynamic programming for finding exact solutions for vertex cover and independent set.
   - Sketch of dynymic programming algorith for finding an optimal vertex colouring.
9. Fixed parameter tractability, reductions to a problem kernel.
   - The Minimum strong spanning subdigraph problem.
   - The feedback vertex set problem.