Institut for Matematik og Datalogi
Syddansk Universitet

May 4, 2011
JBJ

DM515 – Spring 2011 – Weekly Note 5

Handing in obligatory projects
Since Alessandro is away until May 13 we have decided to allow you to hand in the project as late as Thursday May 12 at 3 p.m. Handing in should be done either electronically by sending an email with a PDF of the project to Alessandro (maddaloni@imada.sdu.dk) or putting the report in his mailbox in the secretarys office before 3 p.m thurday. Of course you must remember to put all names of the participants in the project on the report. If you want a receipt for having handed in the project, you should prepare two copies of such a receipt with the names of the participants and hand in the project to me instead at the lecture on May 12. Then I will sign the receipt.

Week 19
We will follow the original program and have 2 lectures and exercises once (wednesday). The extra exercises will be in week 21 instead. In that week you will have exercises monday, wednesday and thursday.

Stuff covered in Week 18:

- A very brief account of rest of Chapter 5 in MG
- MG section 6.3 (I gave the proof of the strong duality theorem).
- Branch and bound Clausen and Larsen Section 9.1. See also Gutin Chapter 6.
- BJG sections 3.1- 3.5 (maxFlowMinCut theorem)

Lecture monday 12-14

- BJG 3.5-3.6.1, 3.10.1 and 3.11.1
- BJG pages 55-58 (shortest paths when there are negative weight arcs and how to find a negative cost cycle).

Lecture Thursday 12-14

Cutting plane methods for TSP: LP-relaxation, 1-tree bound, Comb inequalities, This will be based on pages 252-271 of the book Combinatorial Optimization by Cook, Cunningham, Pulleyblank and Schrijver, Wiley Interscience 1998 (Cook). These pages were be handed out on May 2 (If you are not at that lecture you may get them by coming to my office). Some notation used in (Cook): When $G = (V, E)$ is a graph and $S \subset V$, $\delta(S)$ denotes the set of edges between S and $V - S$ and $\gamma(S)$ denotes the set of edges inside S.

Exercises Wednesday 8-10

These will be handled by Sven Simonsen as Alessandro is away until Friday May 13th

1. Summer 2008 Problem 2 (a)-(d)
2. Summer 2008 Problem 5
3. BJG Problem 3.11.
4. Use augmenting paths to find a maximum (s,t)-flow in the network of Figure 1. Use the resulting maximum flow to identify a minimum cut with the same capacity as the value of the flow.

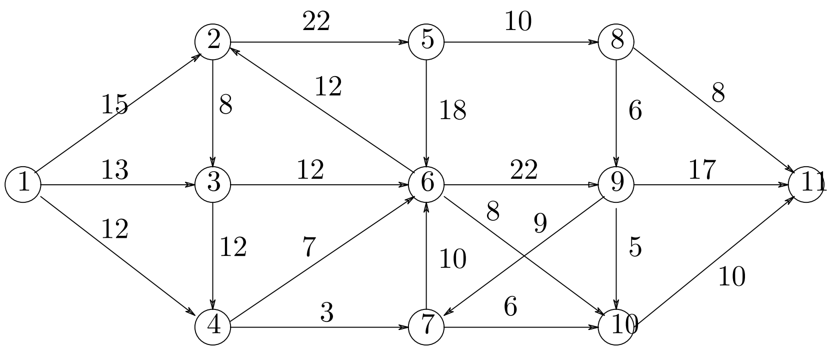

Figure 1: A network with capacities shown and all lower bounds zero. The source s is the vertex 1 and the sink t is the vertex 11.

5. Consider the two bipartite graphs in Figure 2.

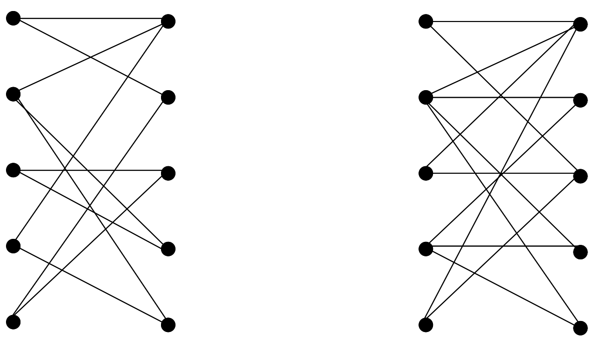

Figure 2: two bipartite graphs

(a) Find a maximum matching in each of the graphs by converting them into flow networks and finding a maximum integer-valued (s, t)-flows as in BG Section 3.11.1. Use the Ford-Fulkerson method (augmenting paths).

(b) Use the resulting maximum (integer-valued) flow to identify a minimum vertex cover for each of the graphs (as in the proof of Theorem 3.11.2 in BG).

(c) In case the maximum matching is smaller than 5 use the final maximum (integer-valued) flow to identify a set of vertices whose set of neighbours is smaller than the set itself (as in the proof of Theorem 3.11.3 in BG).

6. Read Section 2.3.4 in BG on the Bellman-Ford algorithm and be prepared to discuss the correctness of the algorithm. Just as in Dijkstra's algorithm we can maintain, for every vertex $v \neq s$ a predecessor for v on the current shortest path from s to v. These start out being "nil" and when $d(v)$ is changed by relaxing the arc uv the predecessor will become u. Show how to use the predecessor arcs to find a negative cycle in the case the step 3 (bottom of page 56) returns the message that D has such a cycle. Hint: consider what happens when the predecessor graph contains a cycle for the first time (its starts having no arcs).