Institut for Matematik og Datalogi
Syddansk Universitet

May 13, 2011
JBJ

DM515 – Spring 2011 – Weekly Note 6

obligatory projects
Alessandro will correct these and consult with me so that we will have the protocal ready nex thursday (if not before).

Stuff covered in Week 19:

- BJG 3.5-3.6.1, 3.10.1 and 3.11.1
- Pages 261-266 in Combinatorial Optimization by Cook, Cunningham, Pulleyblank and Schrijver, Wiley Interscience 1998 (Cook). These pages (in fact page 252-271) has been handed out during several lectures. IF you still dont have a copy, then come and pick up one from me next week (after that I will be away till after the exam).

Lecture Monday, May 16, 2011, 12-14
MG sections 8.2-8,3

Lecture Thursday, May 19, 2011 12-14

This is the last lecture. I will mainly spend it covering all of the course and perhaps also give some more examples of integer programming formulations and how to solve these.

Exercises Wednesday, May 18, 2011 8-10

1. Questions 6.5.2 and 6.5.9 in Gutin's notes.
2. Another formulation of TSP. Let x_{ij} be a 0-1 variable indicating whether or not vertex j comes immediately after vertex i in the tour (that is, we fix an orientation of the tour so if i is just after j, then j is not just after i, i.e. at most one of x_{ij}, x_{ji} can be 1) and let c_{ij} be the distance. The length of the tour given by x is then

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \tag{1}$$

which we wish to minimize over all x which correspond to a tour (a hamiltonian cycle). Since each vertex is preceeded and followed by exactly one vertex in a tour x must satisfy (2) and (3):

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n \tag{3}$$

The optimal solution to (1)-(3) may still not be a hamiltonian cycle, but it is always a collection of cycles. In fact (1)-(3) describe exactly the assignment problem which you have seen in BG 3.12. In order to force the solution to be just one cycle we add the following sets of conditions:

$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \text{ for every proper subset } S \subset \{1, 2, \ldots, n\} \tag{4}$$

The problem about the formulation above is the there are exponentially many constraints. Now we will look at another formulation (due to Miller, Tucker and Zemlin 1960) which also eliminates subtours and has only a polynomial number of constraints.

(a) Fix vertex 1 to be the home base and for each other vertex i let u_i be an arbitrary real number. Show that if x is a feasible solution to (1)-(4), then we can choose values for $u_2, u_3, \ldots, u_n$ so that the following holds:

$$u_i - u_j + n x_{ij} \leq n - 1, \quad i, j = 2, 3, \ldots, n. \tag{5}$$

Hint: consider the number of edges from vertex 1 to vertex i along the tour corresponding to x and choose u_i based on this.

(b) Show that if x is a 0-1 solution satisfying (2), (3) but violating (4), then (5) cannot hold for all $i, j = 2, 3, \ldots, n$. Hint: consider the sum of these equations along a subtour which does not contain vertex 1.

(c) The observation above shows that (1),(2), (3), (5) and x 0-1 valued is a valid formulation of TSP. Discuss the quality of the LP-relaxation of this formulation compared to the classical one using the subtour constraints.

3. Consider the fractional LP solution to a TSP problem in Figure 1. Find a valid in-equality (one which holds for all 0-1 solutions) which cuts off the LP solution x^* of Figure 1. Hint: Does x^* satisfy the constraints (4)?

4. Consider the fractional LP solution y^* to a TSP problem given in Figure 2. Identify a violated Comb inequality, which when added to the formulation will cut away y^*.

5. Summer 2008 Problem 4.

6. Discuss the problems in the project (if there is no more time this will be done in Week 21).

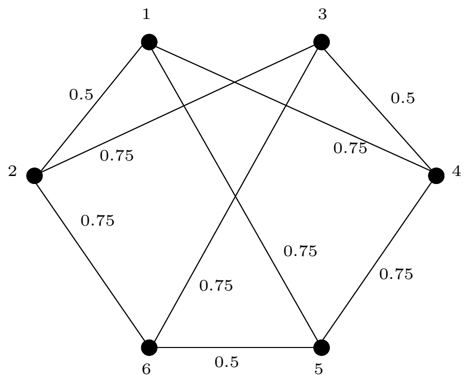

Figure 1: Fractional solution to a TSP problem.

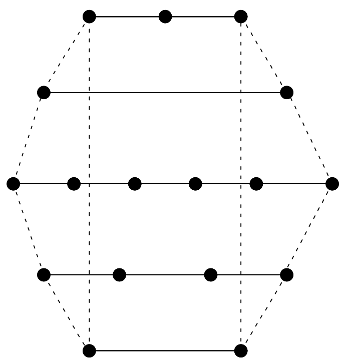

Figure 2: A fractional LP solution y^* to a TSP instance. Dotted lines mean $y* = \frac{1}{2}$ and full lines mean $y^* = 1$.