Problem 1 Given a FA M and $w \in \Sigma^*$ is $w \in L(M)$?

Problem 2 Given FA M is $L(M) = \emptyset$?

Problem 3 Given FA's M_1, M_2, is $L(M_1) \subseteq L(M_2)$?

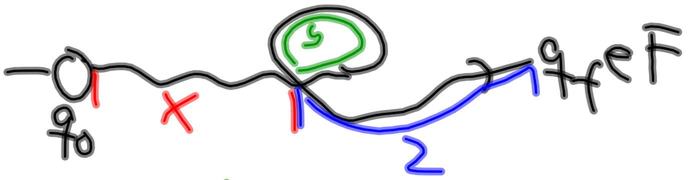
Problem 4 Given FA's M_1, M_2, is $L(M_1) = L(M_2)$?

Problem 5 Given FA M, is $L(M) = \Sigma^*$?

Problem 6 Given regular expr. α, construct M s.t. $L(\alpha) = L(M)$

Problem 7 Given NFA M, Find reg. exp. α, s.t. $L(M) = L(\alpha)$.

L regulært $\Leftrightarrow \exists$ DFA M
så $L = L(M)$
Lad $q = \#$ tilst. i M.
se på w så $|w| \geq p$



$w = xyz$
$|xy| \leq p$
$|y| > 0$

alternativ form:

1. $w = \bar{x}\bar{y}\bar{z}$
2. $|\bar{y}\bar{z}| \leq p$
3. $|\bar{y}| > 0$

Ex 1.77 $E = \{ 0^i 1^j \mid i > j \}$

anvend alternative version (baglæns) af PL

$s = 0^{p+r} 1^p$ hvor p er pumpelgd.

$r \geq 1$

Modstander returnerer x, y, z så

$0^{p+r} 1^p = xyz$, $|yz| \leq p$, $|y| > 0$

se på $xy^{r+1}z$: $0^{p+r} 1^{p+r \cdot |y|} \notin E$ $\rightarrow\!\leftarrow$

<u>Problem 1:</u> · Hvis M er en DFA

Så kør M på w: $w \in L \Leftrightarrow M$ er i en accept state.

· Hvis M er en NFA

– Lav ækv. DFA i exp tid $2^{|Q|} \cdot pol$ og check som ovenfor.

– Simuler M på w: $w = w_1 w_2 \cdots w_n$

$w_i \in \Sigma$

$\sim O(|w||Q_M|^2)$

<u>Problem 2</u> Uanset om M er DFA eller NFA:

$$L(M) \neq \emptyset$$
$$\Updownarrow$$
$$\exists\ (q_0, F)\text{-vej i } M.$$

$$q_0 \rightsquigarrow f \in F$$

<u>Problem 3</u>

$$L(M_1) \subseteq L(M_2)$$

$$\Updownarrow$$

$$\underbrace{L(M_1) \cap \overline{L(M_2)}}_{\substack{= L(M) \\ \text{for en FA } M. \\ \text{brug 2.}}} = \emptyset$$

$S \to aSa \mid bSb \mid A$, $A \to a \mid b \mid \varepsilon$

1. $S_0 \to S$ new variable S_0

2. order variables A, S

   $S \to aSa \mid bSb \mid \varepsilon$, $A \to a \mid b$ (Eliminate $A \to \varepsilon$)

   $S \to aa \mid aSa \mid bb \mid bSb$ (Eliminate $S \to \varepsilon$)

3. $S_0 \to aa \mid aSa \mid bb \mid bSb$

   $S \to aa \mid aSa \mid bb \mid bSb$ Eliminate $S_0 \to S$

   $A \to a \mid b$

4. $S_0 \to aa \mid aV_{Sa} \mid bb \mid bV_{Sb}$

   $S \to aa \mid aU_{Sa} \mid bb \mid bU_{Sb}$

   $V_{Sa} \to Sa$, $V_{Sb} \to Sb$ $A \to a \mid b$

   $U_{Sa} \to Sa$; $U_{Sb} \to Sb$

Final (reduced)

$S_0 \to AV_{Sa} \mid BV_{Sb} \mid AA \mid BB$

$S \to AV_{Sa} \mid BV_{Sb} \mid AA \mid BB$

$V_{Sa} \to SA$

$V_{Sb} \to SB$ $\quad + \underline{S \to \varepsilon}$

$A \to a$

$B \to b$

$$R: \begin{cases} A \rightarrow 0A1 \mid B \\ B \rightarrow \# \end{cases} \qquad G_1$$

$$A \Rightarrow 0A1 \Rightarrow 00A11 \Rightarrow 00B11$$
$$\Rightarrow 00\#11$$

$$G = (V, \Sigma, R, S)$$

$$L(G) = \{ \omega \mid S \overset{*}{\Rightarrow} \omega \}$$

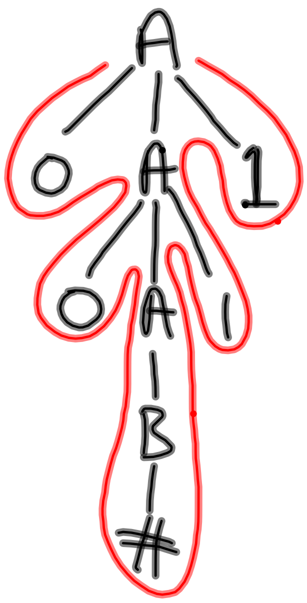
$$L(G_1) = \{ 0^n \# 1^n \mid n \geq 0 \}$$

Hvis L er regulært så er
$L = L(G)$ for en CFG G.

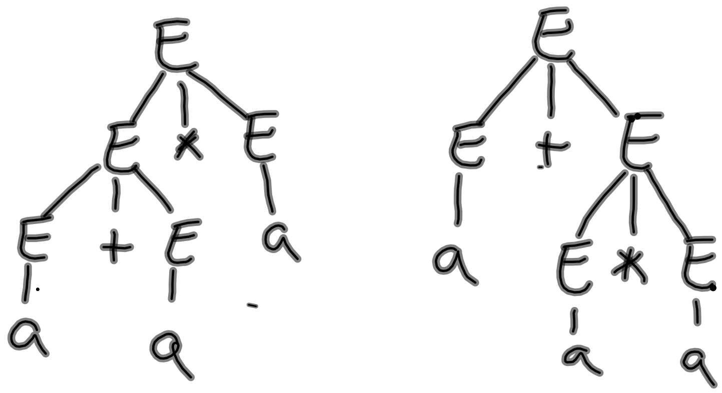
P: Lad $M = (Q, \Sigma, \delta, q_0, F)$ DFA: $L(M) = L$

sæt $V = Q$
$S = q_0$

$R: \{ q_i \longrightarrow a q_j \mid \delta(q_i, a) = q_j \}$
$\cup \{ q_f \rightarrow \varepsilon \mid q_f \in F \}$


A
0
A
1
0
A
1
B
#

$G_5 \quad E \to E+E \mid E*E \mid (E) \mid a$



$a+a*a$

Chomsky normal form.

R: $A \Rightarrow BC \quad B, C \in V - S$

$A \to a \quad a \in \Sigma$

$+ \quad S \to \varepsilon$

<u>Thm</u> $\forall$ CFG G

$\exists$ CNF CFG G': $L(G) = L(G')$

1. Tilføj $S_0 \to S$ S_0 ny var.

2. Fjern ε produktioner ($C \to \varepsilon$)

$R \to uAvAw$

$\downarrow$

$R \to uAvAw \mid uvAw \mid uAvw$

? $B \to A$ ($A \to \varepsilon$)

$B \to \varepsilon$

3. $R \to A$ ($A \to u$)

$\downarrow$

$R \to u$

$A \to u_1 u_2 \cdots u_k \quad k \geq 3 \quad u_i \in V \cup \Sigma$

$A \to u_1 U_2$

$U_2 \to u_2 U_3$

$\vdots$

$U_{k-2} \to u_{k-1} u_k$

? $U_3 \to a U_4$

$U_3 \to A U_4$

$A \to a$