Thm 5.28

$A \leq_m B$ and B Turing-recognizable

$\Downarrow$

A is Turing recognizable

Thm 5.29 $A \leq_m B$ and A not T-recog

$\Downarrow$

B not T-recog.

Note $A \leq_m B$

$\Updownarrow$

$\overline{A} \leq_m \overline{B}$

Thm 5.30 Neither EQ_{TM} nor $\overline{EQ_{TM}}$ are T-recognizable

P: $A_{TM} \leq_m \overline{EQ_{TM}}$

$L(M_1) = \emptyset$

$$L(M_2) = \begin{cases} \emptyset & M \text{ does not acc } w \\ \Sigma^* & M \text{ acc } w \end{cases}$$

$\langle M, w\rangle \xrightarrow{f} \langle M_1, M_2\rangle$

$\langle M_1, M_2\rangle \in \overline{EQ_{TM}} \iff \langle M, w\rangle \in A_{TM}$

so $A_{TM} \leq_m \overline{EQ_{TM}}$

known not to be T-recog $\Downarrow$ $\overline{A_{TM}} \leq_m EQ_{TM}$

Thus EQ_{TM} is not T-recognizable.

$$L(M_1) = \Sigma^*$$

$$L(M_2) = \begin{cases} \emptyset & \text{if } M \text{ does not acc } w \\ \Sigma^* & M \text{ acc } w \end{cases}$$

$$\langle M_1, M_2 \rangle \in EQ_{TM} \iff M \text{ acc } w$$

so $A_{TM} \leq_m EQ_{TM}$

$$\Downarrow$$

$\overline{A_{TM}} \leq_m \overline{EQ_{TM}}$

Thus $\overline{EQ_{TM}}$ is not T-recognizable.

Jan 2010 Problem 6

1. Given $\langle M \rangle$ is $L(M) = L_4$?

Undecidable via Rice:

In 4(b) we "saw" a TM M_1 with $L(M_1) = L_4$

$M_\emptyset$ has $L(M_\emptyset) = \emptyset$

so the property:

$L(M) = L_4$ is non-trivial

and Rice's thm can be applied

2. Given $\langle M_1\rangle \langle M_2\rangle$
is $L(M_1) \subseteq L(M_2)$? R?

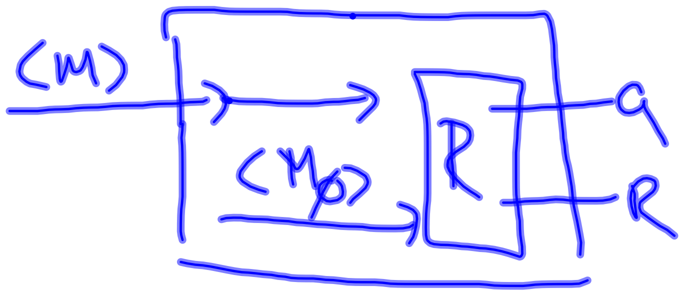
Undecidable:

Take $M_2 = M_\emptyset$

now $L(M_1) \subseteq L(M_2)$

$\Updownarrow$

$L(M_1) = \emptyset$ undecidable



decides E_{TM}

where R is the assumed TM

deciding the question 2)

3. decidable

check state table of M.

4. Given $\langle M \rangle$ does $\exists$ regular L st

$$L(M) \cap L = \emptyset \ ?$$

decidable Take $L = \emptyset$ regular

5. Given $\langle M_1 \rangle, \langle M_2 \rangle$

Q: will both M_1 and M_2 take ≥ 42 steps on ϵ?

decidable: Simulate each for 41 steps and reject if one of M_1, M_2 has stopped otherwise accept

6) Given $\langle M \rangle$ and $\langle w \rangle$

Q: $\exists ?$ $k \geq 1$ s.t $w^k \in L(M)$?

Undecidable by Rice's thm

$L(M_1) = \{ w^k \mid k \geq 1 \}$ assume w is given

has the desired property

$L(M_2) = \emptyset$ does not

so non-trivial property

If w cannot be assumed given

take $L(M_1) = \Sigma^*$ $L(M_2) = \emptyset$

October 2010 Prb 5

1. $\langle M \rangle$ is $L(M)$ non-reg.?

Undecidable by Rice

$L(M_1) = \{a^n b^n \mid n \geq 0\}$

$L(M_2) = \Sigma^*$

2. $\langle M \rangle, \langle w \rangle$ does M have $|w|$ states?

dec. look at $\langle M \rangle$ and count # states.

3. $\langle M_1 \rangle, \langle M_2 \rangle$

Q: $\exists$? L s.t

$L \cap L(M_1) \subseteq L(M_2)$?

dec take $L = \emptyset$

4. $\langle M \rangle, \langle w \rangle$

Q: does M make $\leq |w|$ steps on w?

dec. simulate M on w for up to $|w|$ steps.

If M stopped say 'Yes'

otherwise say 'No'

6. <M>, k

Q: is $|L(M)| = k$?

Undecidable by Rice:

$L(M_1) = \{w_1, w_2, \ldots, w_k\}$

for some fixed set of k strings

$L(M_2) = \Sigma^*$

Jan 2008, 5

(1) $\langle M \rangle$ M halts on two strings of different lengths.

(1)' $\langle M \rangle$ L(M) contains two string of diff length.

(1)' is undec. by Rice

Given M make M' which loops if M rejects

$\langle M \rangle \in (1)' \Leftrightarrow \langle M' \rangle \in (1)$

(1)' is undec. by Rice

$\Downarrow$ (1) undec.

2. skip difficult

3. $\{\langle M \rangle \mid L(M) \text{ is a CFL}\}$

undec. by Rice.

$L(M_1) = \Sigma^*$ CFL

$L(M_2) = \{a^n b^n c^n \mid n \geq 0\}$ not CFL

4. $\{\langle M_1\rangle\langle M_2\rangle \mid M_1$ halts on more strings than $M_2\}$

Undec.:

M_2 : T.M that never halts

$\langle M\rangle \longrightarrow \langle M'\rangle$

M' halts exactly when M would acc.

$L(M) \neq \emptyset$

$\Updownarrow$

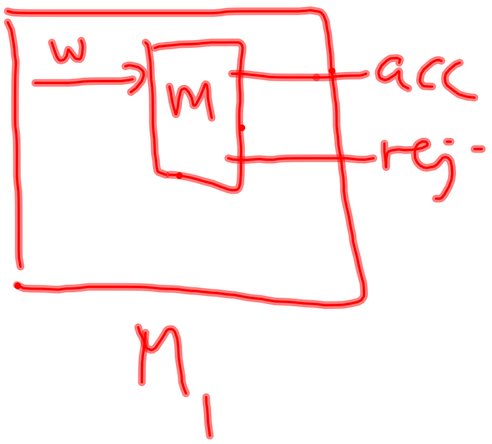
M' halts on at least one string

$\Updownarrow$

M' halts on more strings than M_2

(5) $\{\langle M_1\rangle\langle M_2\rangle\langle w\rangle \mid M_1$ halts before M_2 on $w\}$

Undecidable



M_2 is M_1 + some extra steps at the end.

Then: M_1 halts before M_2 on w iff M halts on w