$L = \emptyset$

ε is a string

a language is a set of strings over some alphabet Σ
$(w \in \Sigma^*)$

$G: \quad S \to aSb \mid \varepsilon$

$S \overset{*}{\Rightarrow} \varepsilon$

so $\varepsilon \in L(G)$

$S \Rightarrow aSb \Rightarrow a^2Sb^2 \Rightarrow a^2b^2$

$S \overset{*}{\Rightarrow} uSv$

$G: \; S \to T \mid T' \qquad T, T'$ variables

$T'' \to a \;,\; T \to T' \;,\; T' \to T$

$S \Rightarrow T$

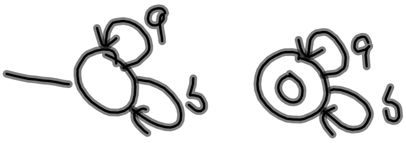
$L(G) = \varnothing$

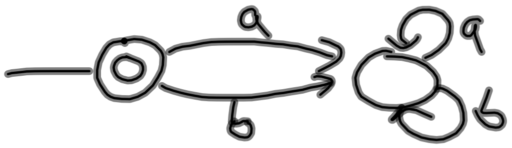
Example for FA's (M)

$L(M) = \emptyset$

$\Updownarrow$

$\nexists$ path from starting state (q_0) to a final state (F)

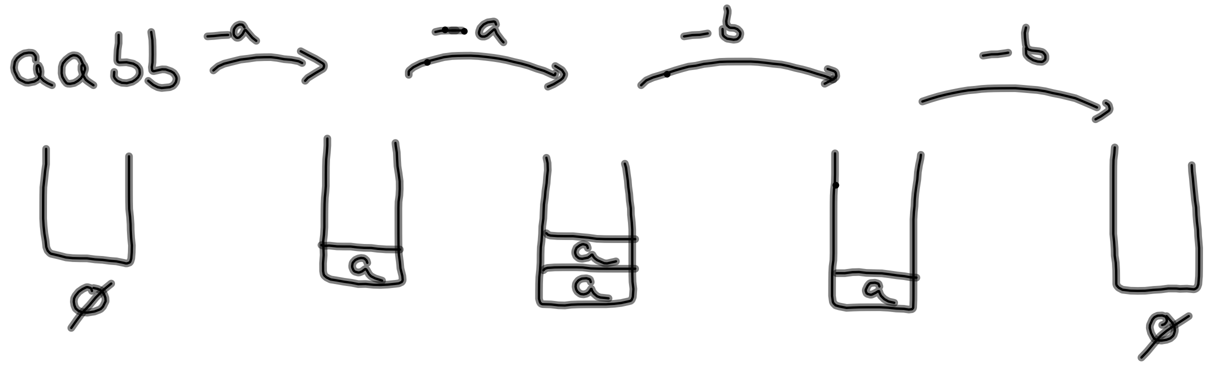


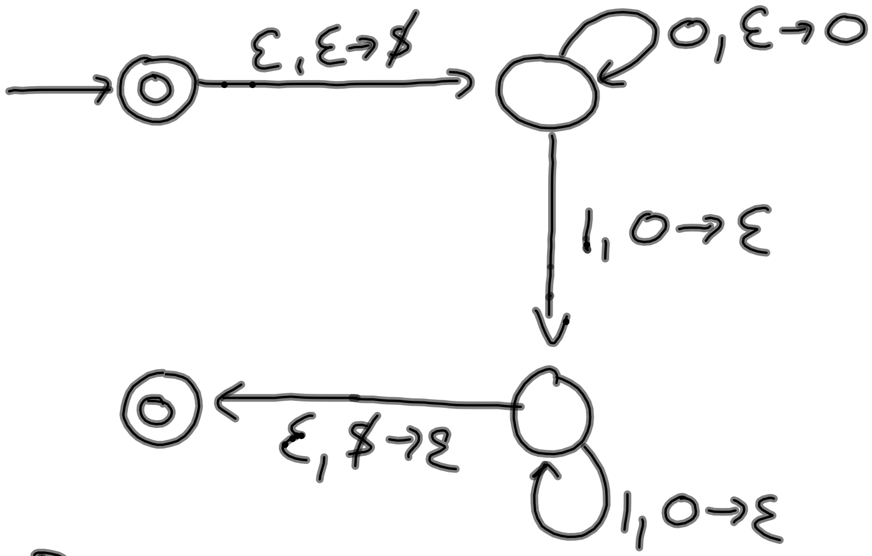


$L = \{\varepsilon\}$

A PDA is a FA together with a stack.

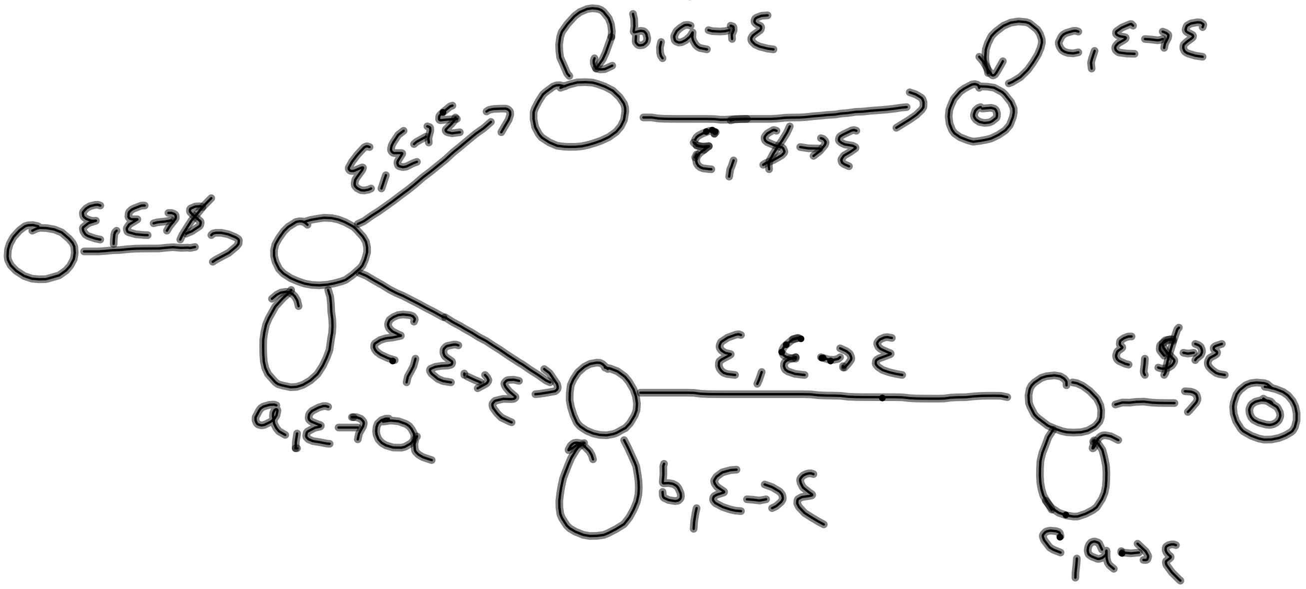
Example $L = \{a^n b^n \mid n \geq 0\}$





PDA for $L = \{0^n 1^n \mid n \geq 0\}$

$L = \{ a^i b^j c^k \mid i, j, k \geq 0,\ i = j \text{ or } i = k \}$



$$\emptyset^* = \{\varepsilon\}$$

$$L^* = \{ w \in \Sigma^* \mid w = w_1 w_2 \cdots w_k \text{ for some } k \geq 0 \text{ and } w_i \in L \}$$

$$S \overset{*}{\Rightarrow} uAv \qquad u,v \in (V \cup \Sigma)^*$$

$$A \in V$$

replace A

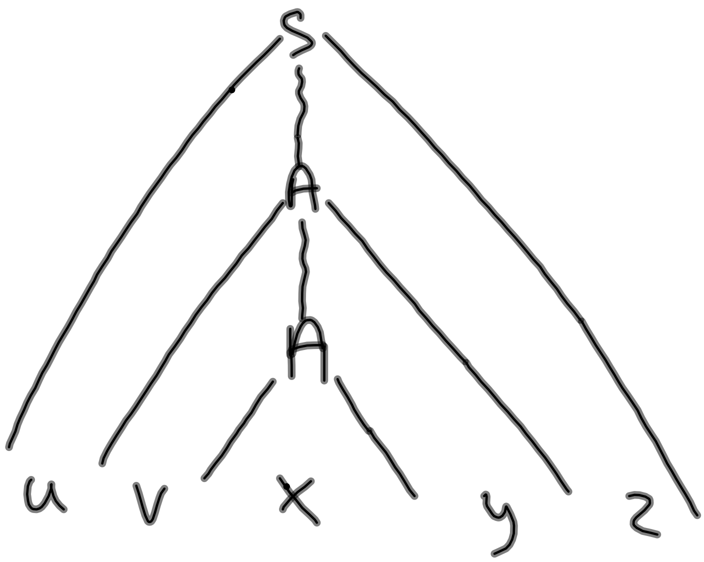
by any productio $A \to \beta$ in R

$|\beta| \leq$ some constant

If $w \in L(G)$ and is "long enough" then every derivation of w will have something like

$A \overset{*}{\Rightarrow} uAv \Rightarrow u'Av'$ in it

u prefix of u
v suffix of v



$w = uvxyz$

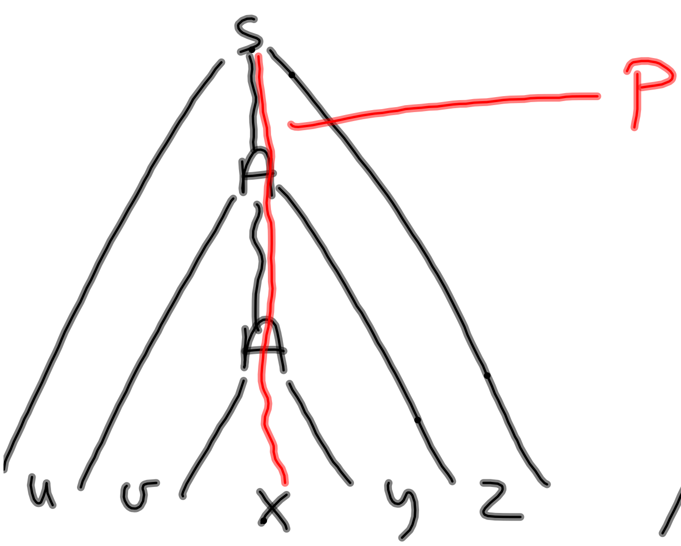
Thm 2.34 PL for CFL

$\forall$ CFL L $\exists p = p(L) \in \mathbb{N}$

s.t $\forall s \in L : |s| \geq p$

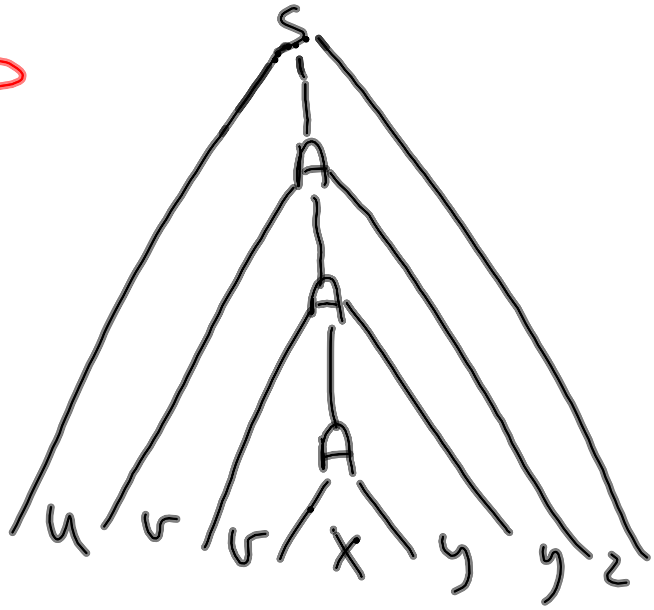
$\exists u, v, x, y, z \in \Sigma^*$ s.t $s = uvxyz$

1. $uv^ixy^iz \in L \ \forall i \geq 0$
2. $|vy| > 0$
3. $|vxy| \leq p$



used $A \overset{*}{\Rightarrow} vAy$

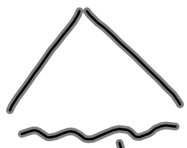
$A \overset{*}{\Rightarrow} x$



Let $b = \max\{2, |\text{longest right hand side of rule in } R|\}$

$p := b^{|V|+1}$

max $|w|$ s.t. w is the "yield" of a parse tree T of height h is b^h



$\leq b^h$

Conclusion: since $|s| \ge p = b^{|V|+1}$

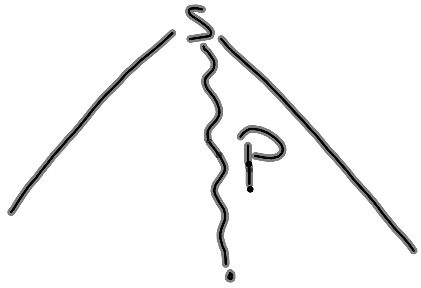
Every parsetree $\mathcal{T}$ that "yields" s

contains a path P of length (#edges)

at least $V+1$

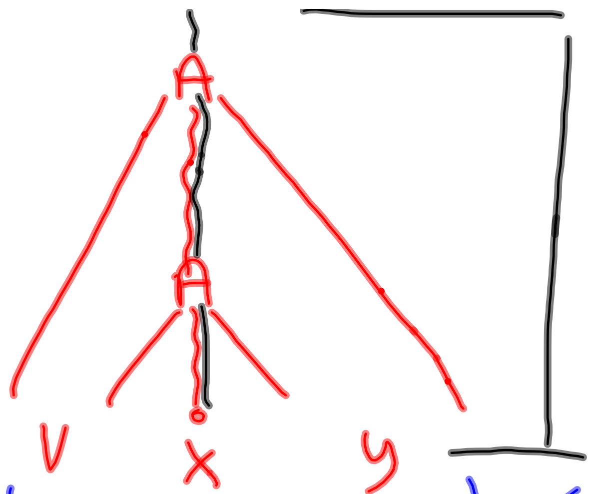
so some variable

A is repeated on P



- may assume that T is minimal in terms of # nodes
- A may be chosen among the last path of P consisting of the last $|V|+1$ edges.

so 2. holds



last $|V|+1$ edges of P

$|vxy| \le b^{|V|+1} = p \Rightarrow 3.$

$B = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL

Pf: suppose $B = L(G)$ for some G
and let $p = p(G)$ (p from PL)
- take $s = a^p b^p c^p$
- adversary splits s into $s = uvxyz$
s.t 1., 2. and 3. of PL hold

$s = a^p b^p c^p$ $\quad a \cdots a\, b \cdots b\, c \cdots c$

possible locations of substring vxy

(since $|vxy| \le p$)

If , , hold then $uxz \notin L$

If , then $uv^2xy^2z \notin L$