Given PDA M (restricted PDA)

Form CFG:

$$V = \{ A_{pq} \mid p,q \in Q(M) \}$$

$$S = A_{q_0 q_{accept}}$$

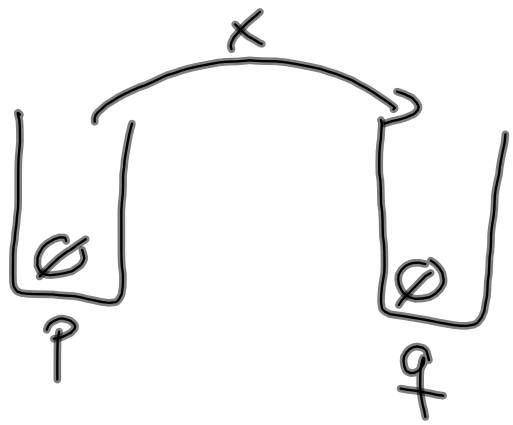
$\forall p,q,r,s \in Q(M), t \in \Gamma, a,b \in \Sigma_\varepsilon$

If $(r,t) \in \delta(p,a,\varepsilon) \wedge (q,\varepsilon) \in \delta(s,b,t)$

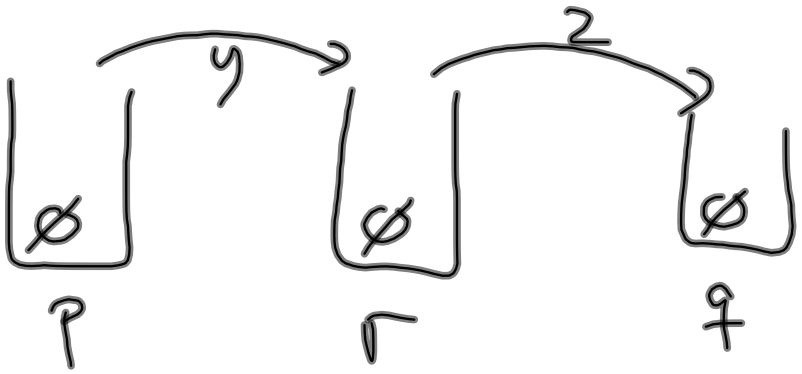
then add rule $A_{pq} \to a A_{rs} b$

Two possibilities for M

when



1. no empty stack in between
(on prefix of x)

2. 

$\forall_{p,q,r} \in Q$ add rule

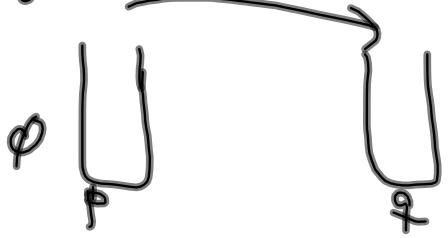
$A_{pq} \to A_{pr} A_{rq}$,

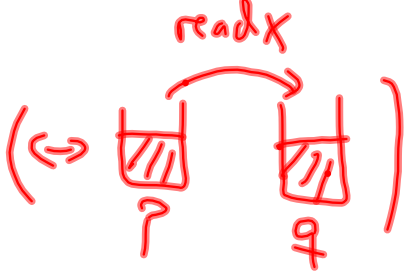
$\forall_p \in Q$ add rule

$A_{pp} \to \varepsilon$

Claim $A_{pq} \overset{*}{\Rightarrow} x \quad x \in \Sigma^*$

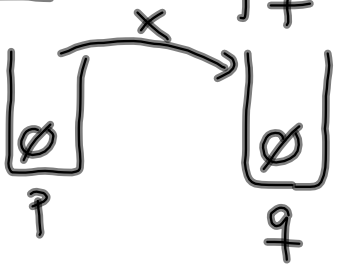
$\Updownarrow$ M reads x





Claim 2.30 If $A_{pq} \overset{x}{\Rightarrow} x$

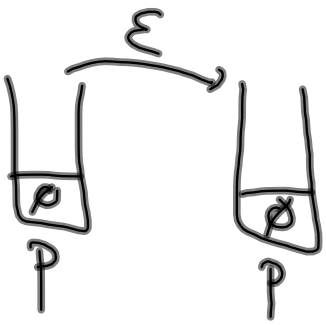
then



Pf: Induction on # steps in derivation.

1 step: $p=q$ and we used $A_{pp} \to \varepsilon$

clearly



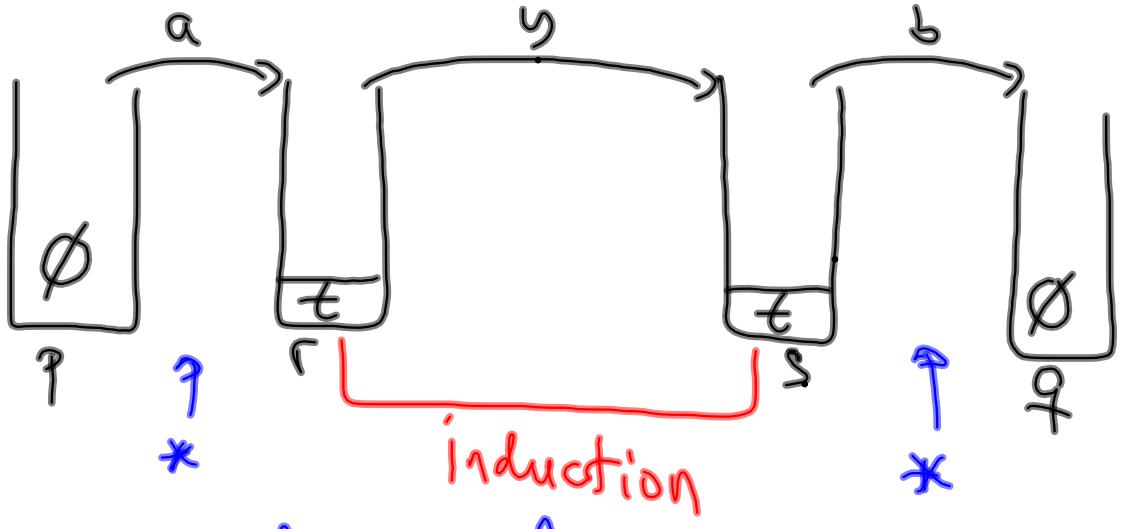
assume ok if derivation has $\leq k$ steps.

$k+1$ steps : 2 cases

a) $A_{pq} \to a A_{rs} b$ first step

b) $A_{pq} \to A_{pr} A_{rs}$

a): $x = ayb$ for some $y \in \Sigma^*$ and $A_{rs} \overset{*}{\Rightarrow} y$ in k steps



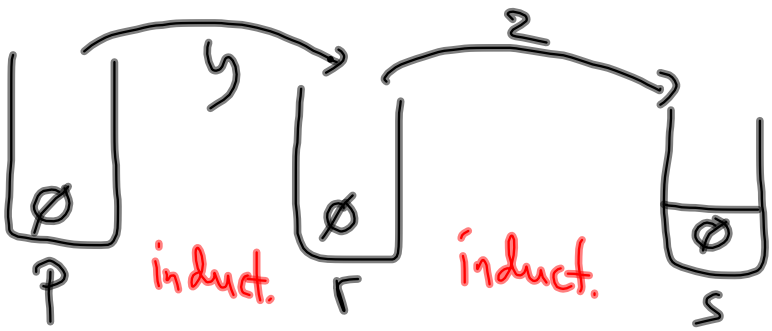
Since $A_{pq} \to aA_{rs}b$ was added as a rule

b): $A_{pq} \rightarrow A_{pr} A_{rq}$

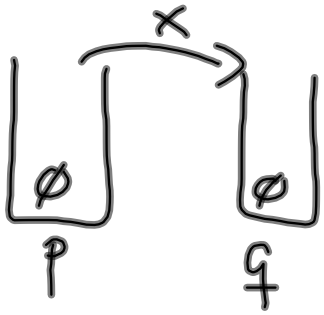
so $x = yz$

where $A_{pr} \overset{*}{\Rightarrow} y$ in $< k$ steps

$A_{rq} \Rightarrow z$ - - - - - - - - -



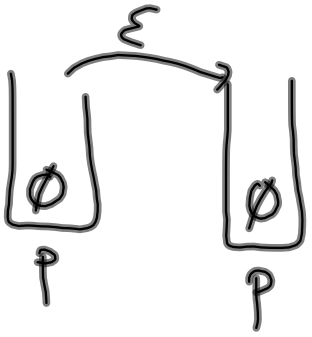
Claim 2.31



$\Rightarrow A_{pq} \overset{x}{\Rightarrow} x$

pf by induction on # steps made by M.

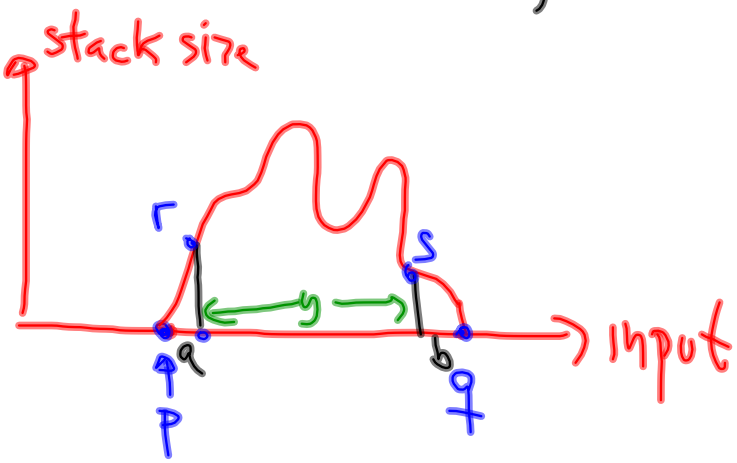
0 steps



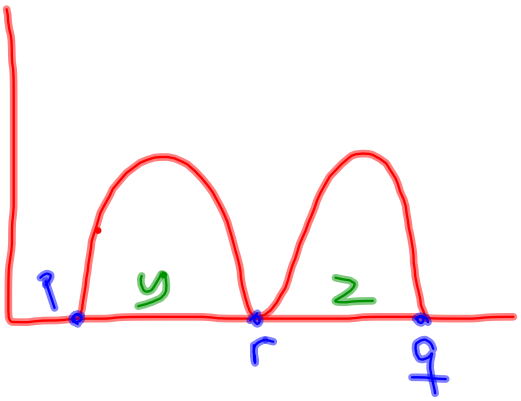
$A_{pp} \to \varepsilon \; V'$

assume ok when M takes $\leq k$ steps and consider the case when M takes $k+1$ steps

Case a



Case b



In case b $A_{pr} \Rightarrow y$, $A_{rq} \Rightarrow z$ } by induction

So $A_{pq} \Rightarrow A_{pr} A_{rs} \Rightarrow yz = x$

Case a $A_{pq} \to a A_{rs} b$

(by the steps that M took)

and by induction

$A_{rs} \overset{*}{\Rightarrow} y$ so

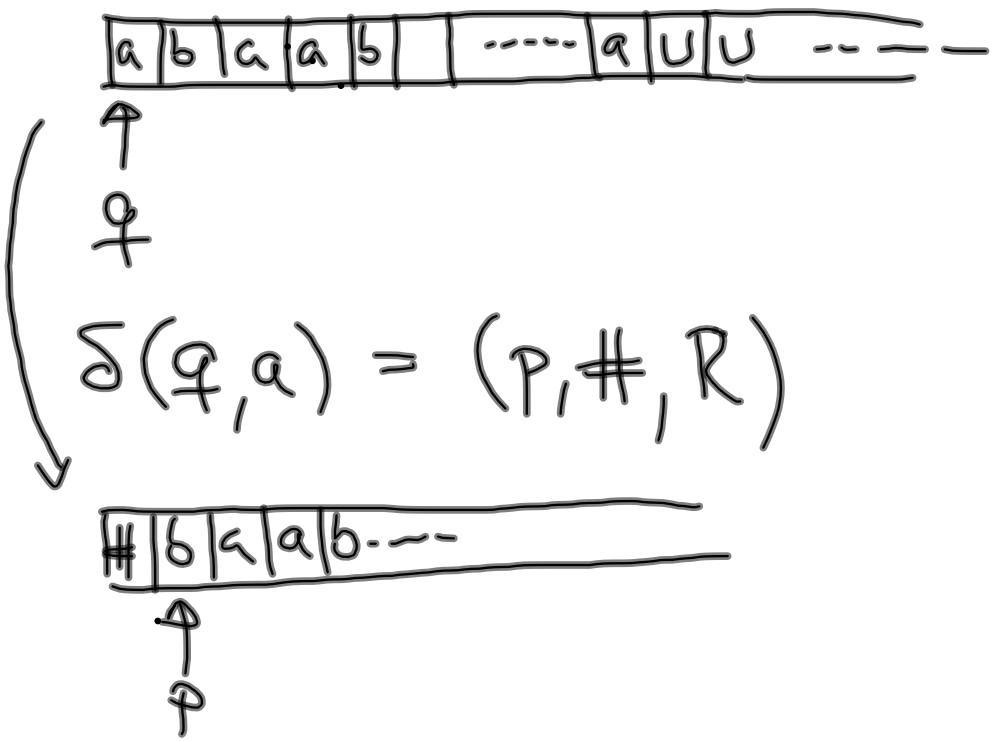
$A_{pq} \Rightarrow x A_{rs} b \overset{*}{\Rightarrow} ayb = x$

Conclusion

$$S = A_{q_0 q_{accept}} \overset{*}{\Rightarrow} x$$

$$\Updownarrow$$

$$x \in L(M)$$


a b c a b ----- a U U
q
δ(q, a) = (p, #, R)
b c a b
p

$$\{ w\#w \mid w \in \{0,1\}^* \}$$

0 1 1 # 0 1 1 U U · ·

x 1 1 # 0 1 1

x 1 1 # 0 1 1

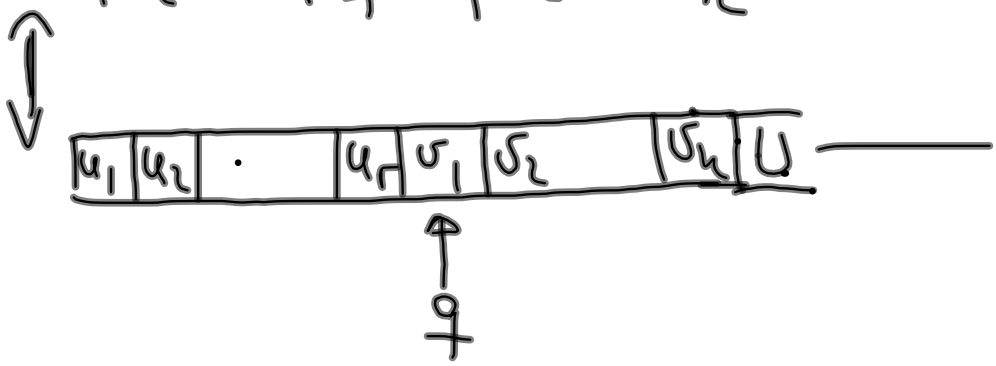
x 1 1 # 0 1 1

x 1 1 # x 1 1

x 1 1 # x 1 1

Configuration of TM M.

$u_1 u_2 \cdots u_r q v_1 v_2 \cdots v_k$



Starting configuration on $w \in \Sigma^*$

$q_0 w \quad \longleftrightarrow$

Accepting config

$u\, q_{accept}\, v$

Def M accepts w

$\Updownarrow$

$\exists$ configs $C_1, C_2, \ldots, C_k$

$C_1 = q_0 w$

$C_i \to C_{i+1} \quad i \le k-1$

$C_k = u q_{acc} v$

$L(M) = \{ w \mid M \text{ accepts } w \}$

Def 3.5

L is Turing-recognizable

$\Downarrow$

$L = L(M)$ for some TM M

If M always halts then M is called a decider.

L is Turing-decidable

$\Downarrow$

$L = L(M)$ for some recognizer M.

$A = \{ 0^{2^n} \mid n \geq 0 \}$

0000 is good

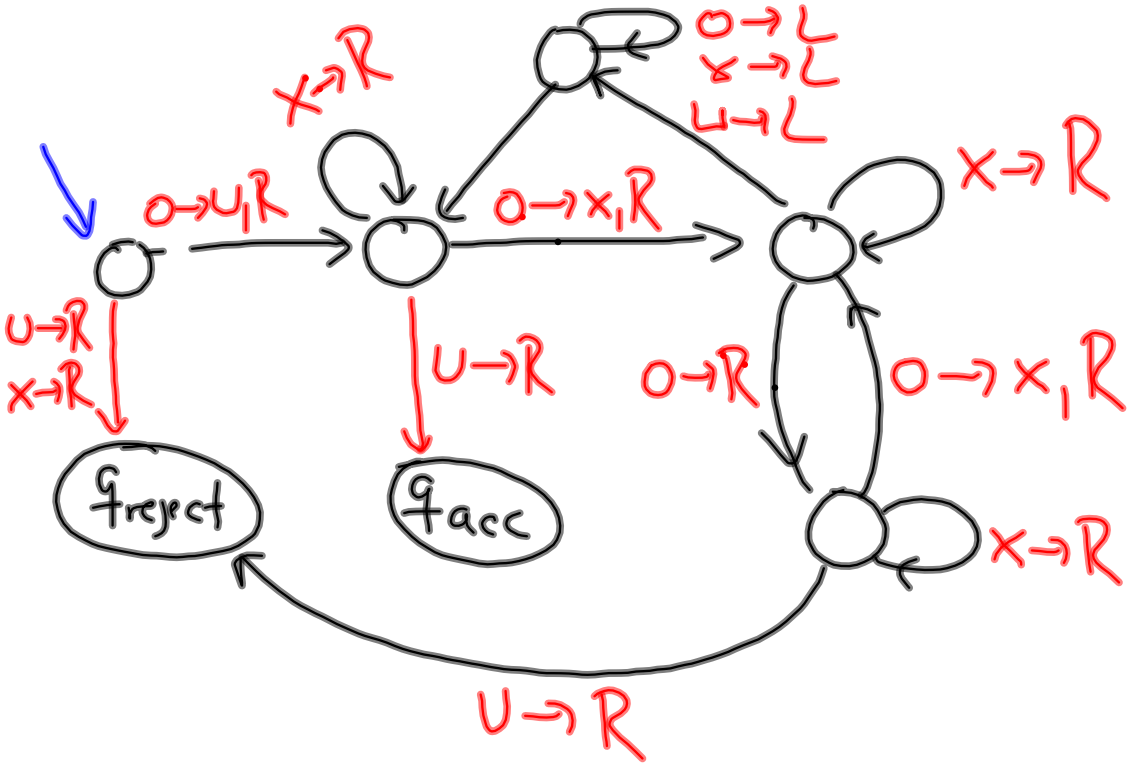
000 is bad

0 ~~0~~ 0 ~~0~~ ~~0~~

q

one left $\Rightarrow$ accept

~~0~~ 0 reject since odd length


0→L
x→L
␣→L
x→R
0→␣,R
0→x,R
x→R
␣→R
x→R
␣→R
0→R
0→x,R
x→R
␣→R
q_reject
q_acc

Some usefull TM's

Shift right

$q_0 w \rightarrow q_0 \triangleright w \quad (w \neq \emptyset)$

1. Mark w_1 as special $\dot{w}_1$
2. Move right until '$\sqcup$'
3. Move Left, read char, write '$\sqcup$'
4. Move right, wrt char, move left
   goto 3.

Exception: if in 3 we read $\dot{w}_1$
wrt $\triangleright$ and stop

Notes (observations)

1. never touched 'left end'
2. could also do
   $q_0 w \to q_0 \sqcup w$
   $q_0 w \to q_0 \# w$
3. $w q w' \to w \# w'$

Left shifting

$q\,\#u_1u_2\cdots u_n \rightarrow q\,u_1u_2\cdots u_n$

1. Move right and read symbol(x)
2. If $x = 'U'$
   Move left, write 'U', stop
3. If $x \neq 'U'$
   Move left, write x
   Move right
   goto 1.

also works

$w \# w' \to ww'$

Copy $w \to ww$

1. $w_1 w_2 \cdots w_n \to w_1 \cdots w_n \#$
2. $w_1 w_2 \cdots w_n \# \to w \# w$
3. Shift left