<u>Thm</u> Let L, L' be regular languages then each the following are regular.

1) $L \cup L'$
2) $\overline{L}$
3) $L \cap L' = \overline{\overline{L} \cup \overline{L'}}$
4) $L - L' = L \cap \overline{L'}$
5) $L \circ L'$
6) L^*

Def 1.52 R is a regular expression over Σ if R is one of the following

1. a for some $a \in \Sigma$
2. ε
3. $\emptyset$
4. $(R_1 \cup R_2)$ where R_i is regular expr over Σ $i=1,2$
5. $(R_1 \circ R_2)$ — || —
6. (R_1^*)

( ) can omitted when we respect order

$* > \circ > \cup$

$a^*ba \cup ab$

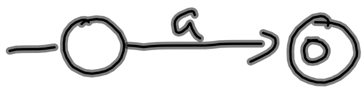
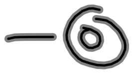
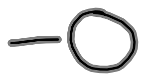
$R^+ = R \circ R^*$

$R^* = R^+ \cup \varepsilon$

$L(R) =$ language generated by R

L 1.55 If $L = L(R)$ then L is regular

i.e $\exists$ (D)FA M s.t $L = L(M)$

P:
- $R = a \ (\in \Sigma)$  NFA
- $R = \varepsilon$ 
- $R = \emptyset$ 
- $R = R_1 \cup R_2 \rightarrow$ use NFA for union of M_1, M_2
- $R_1 = R_1 \circ R_2 \longrightarrow$ ----- concatenation of M_1, M_2
- $R_1^* \rightarrow$ 'NFA for M_1'

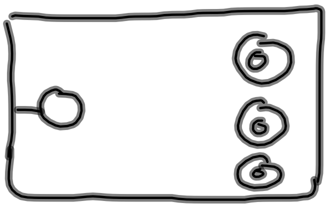
NB: Every finite language is regular!

$L = \{ w_1, w_2, \ldots w_n \mid w_i \in \Sigma^* \}$

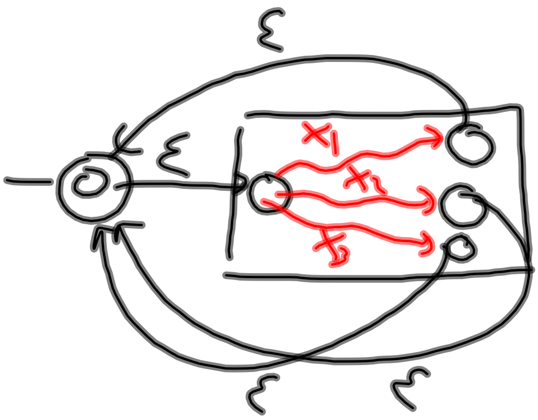
e.g. $w = abaabb$

Corollary Every non-regular language contains arbitrarily long strings.

Correct NFA for L^*



M for L



$x_1 x_2 x_3 \quad x_i \in L$

Convert

input GNFA G with $k \geq 3$ states

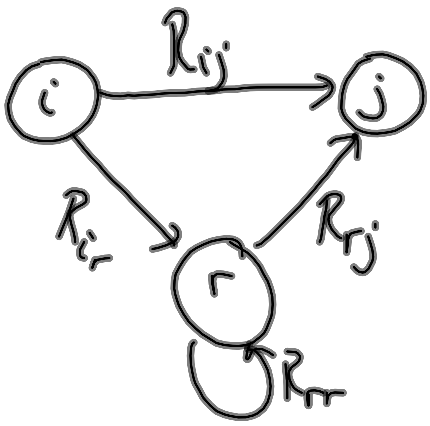
out GNFA G' with $k-1$ states

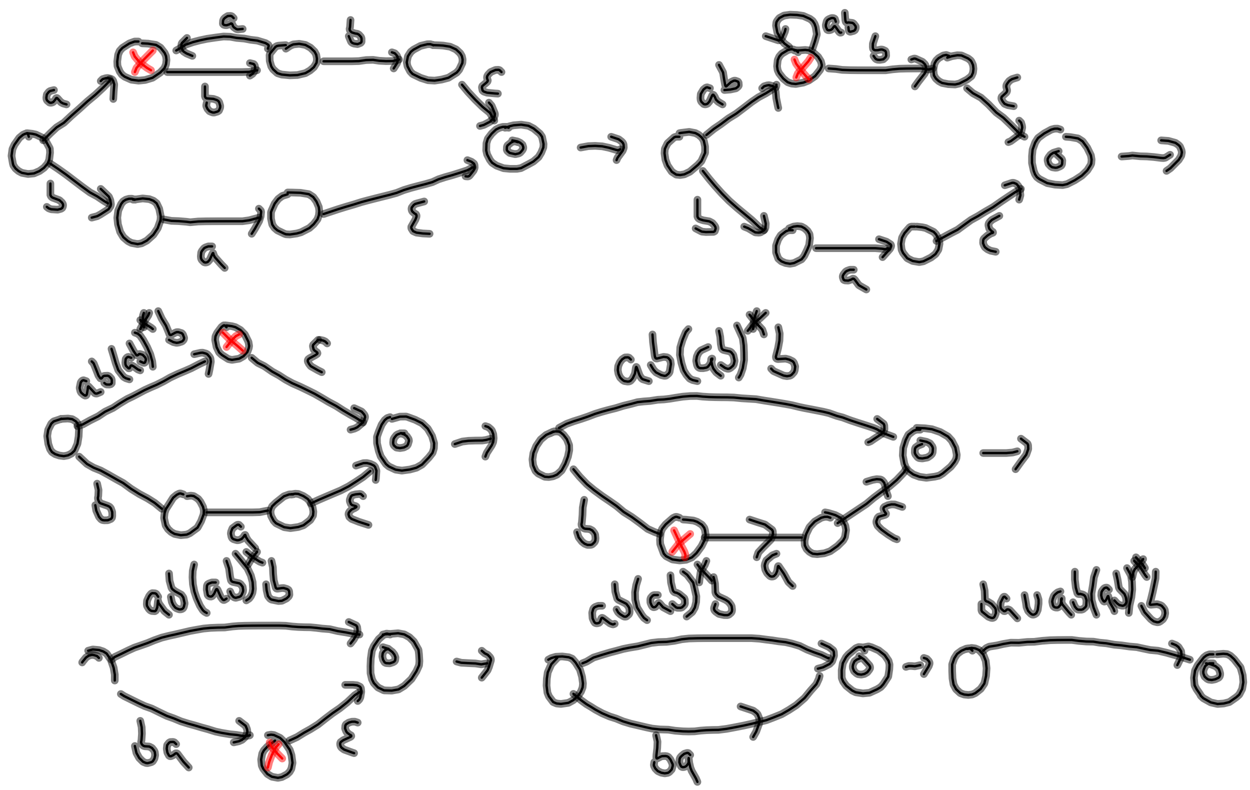
s.t $L(G) = L(G')$

$\forall\, i,j \qquad i \xrightarrow{R_{ij}} j \qquad i,j \neq r$

selected state

$$R_{ij} := R_{ij} \cup R_{ir} R_{rr}^{*} R_{rj}$$




a
b
a
b
ε
b
a
ε
ab
ab
b
ε
b
a
ε
ab(ab)*b
ε
b
a
ε
ab(ab)*b
b
a
ε
ab(ab)*b
ba
ε
ab(ab)*b
ba
ba ∪ ab(ab)*b

Thm 1.70 Pumping lemma

Let A be a regular language.
Then $\exists p \in \mathbb{N}$ so the following holds $\forall s \in A$ where $|s| \geq p$

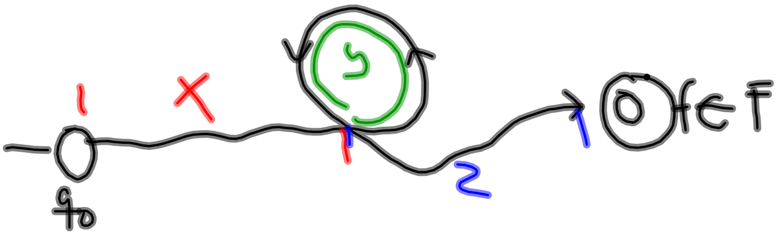
$\exists x, y, z \in \Sigma^*$

1. $s = xyz$
2. $xy^iz \in A \ \forall i \geq 0$
3. $|xy| \leq p$. $|y| > 0$

pf Since A is regular we can take DFA $M = (Q, \Sigma, \delta, q_0, F)$ s.t $L(M) = A$

Set $p := |Q|$

Let $s \in A$ with $|s| \geq p$



(q_0, f)-walk must cross itself!

$L_1 = \{a^n b^n \mid n \geq 0\}$ is not regular

pf: Suppose L_1 was regular.

Let M be DFA $L(M) = L_1$

and $p = $ #states in M

look at $s = a^p b^p \in L_1$

adversary chooses $x, y, z \in \Sigma^*$

s.t. $s = xyz$ and 2.+3. hold.

Now (no matter which choice (s)he made)

$xy \in a^r$ for some r since $|xy| \leq p$

$\Rightarrow a^{p-|y|} b^p \in L_1$ by PL $\rightarrow\leftarrow$

$L_2 = \{ w \in \{a,b\}^* \mid \#a = \#b \}$

so $L_1 = L_2 \cap a^*b^*$

Since a^*b^* is regular
and regular languages are closed
under '$\cap$'
We get that L_1 is regular
if L_2 is regular
But L_1 is <u>not</u> regular so neither is L_2

$L = \{ 0^n \mid n \text{ is a prime} \}$

Claim L is not regular

suppose it is and that $L = L(M)$ for a DFA with p states.

Choose a prime $m \geq p$ and let $s = 0^m \in L$

adversary $0^m = xyz$ s.t 1., 2., 3. in PL hold.

$|xy^iz| = (m-q) + iq$ where $q = |y|$

take $i = m+2q+3$

$|xy^iz| = m + (m+2q+2)q = (m+2q)(q+1)$ not prime!