Institut for Matematik & Datalogi
Syddansk Universitet

November 5, 2007
JBJ

DM517 — Weekly note 1

Welcome to DM517!

The weekly notes and the lectures will be in English as we have at least one foreign participant. If you have any problem understanding what I write or say, please do not hesitate to ask for a danish translation.

The weekly not will be published on the course web page every Friday afternoon. It will contain information about what will happen in the next week (lecture(s) and exercise classes).

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Litterature

Efim Kimber and Carl Smith, Theory of Computing: A gentle introduction.
Besides this book you may look at the following which will be in on the course shelf in the library

- Lewis and Papadimitriou, Elements of the theory of computation, 2nd ed. Prentice Hall, 1998.
- Sipser, Introduction to the theory of computation, PWS publishing company, 1997.

Exam

Written exam January 7 2008.

Lectures:

(1) Tuesday 8-10 in U2 weeks (45-51).
(2) Thursday 12-14 in U20 in weeks (45,47,49,50).

Exercise classes:

(1) Friday 10-12 in U37 in weeks (45-51)
(2) Wednesday 12-14 in U37 in week 46
(3) Thursday 12-14 in U20 in weeks (48,51)

Your instructor is Anna Rasmussen, email ruvimbo@imada.sdu.dk

Lecture November 6, 2007:

(1) Short introduction to the course. You are expected to read Chapter 1 yourself.
(2) Deterministic Finite Automata 2.1
(3) Non-deterministic Finite Automata 2.2
(4) Equivalence of NFAs and DFAs 2.3

Lecture November 8, 2007:

(1) Regular expressions and languages 2.4
(2) Non-regular languages 2.5

Exercises November 9, 2007:

(1) You should do all the exercises 2.1-2.6 pages 54-55. The instructor will select a subset of these to discuss with you.
(2) 2.11 page 58.
(3) 2.12 page 59.
(4) Solve the following problem: :

    A man is travelling with a wolf (w) and a goat (g). He also brings along a nice big cabbage (c). He encounters a small river which he must cross to continue his travel. Fortunately, there is a small boat at the shore which he can use. However, the boat is so small that the man cannot bring more than himself and exactly one more item along (from w, g, c). The man knows that if left alone with the goat, the wolf will surely eat it and the goat if left alone with the cabbage will also surely eat that. The man's task is hence to device a transportation scheme in which, at any time, at most one item from w, g, c is in the boat and the result is that they all crossed the river and can continue unharmed.

    (a) Describe a solution to the problem which satisfies the rules of the "game". You may use your answer to (b) to find a solution.
    (b) Consider strings over the alphabet $\Sigma = \{m, w, g, c\}$ and interpret these as follows: The symbol m means that the man crosses the river alone, w means that he brings the wolf etc.

        Design a finite automaton which accepts precisely those strings over Σ which correspond to a transportation sequence where everybody survives and is legal in the sense that the man can only bring an item (e.g. w) back across the river if it was actually on the shore where the boat just left from. For example $gmcg$ is a legal string (it is not a solution) whereas gc is not legal.

Training exercises:

The exercises below are from the previous version of this course (DM17) which was taught by Klaus Meer. You should use these exercises to refresh definitions and proof techniques. You don't have to do all of them now, but use them when you have time to refresh the notation and basic results that we will use.

As a warm up for the course we like to recall some basic things studied in earlier courses. We shall, however, focus on the way they are used in DM517. Therefore, we start with one of the most fundamental definitions needed in DM517, that of a finite alphabet and strings over such an alphabet.

Definition: A finite set $\Sigma := \{a_1, \ldots, a_s\}$ is called a *finite alphabet*. A *word* or *string* x over Σ is a sequence $x_1 x_2 \ldots x_n$, where all $x_i \in \Sigma$. We say that x has length $n \in \mathbb{N}$ and denote the length by $|x| := n$. For two words $x := x_1 \ldots x_n$ and $y := y_1 \ldots y_m$ we define their *concatenation* $xy := x_1 \ldots x_n y_1 \ldots y_m$.

We also define a (unique) word e of length 0 and call it *the empty word* over Σ.

Finally, the set of all words over Σ is denoted by Σ^*.

Problem 1: Consider the finite alphabet $\Sigma := \{0, 1\}$ and define a function $f : \Sigma^* \to \Sigma^*$ recursively as follows:

$$f(e) := e$$

$$f(x0) := f(x)01 \text{ for any } x \in \{0,1\}^*$$

$$f(x1) := f(x)10 \text{ for any } x \in \{0,1\}^*$$

a) Show that all strings $f(x)$ have the same number of 0s and 1s.

b) Define the set S as the image of the function f, i.e. $S := \{y \in \Sigma^* \mid \exists x \in \Sigma^* \ f(x) = y\}$.

   Give a recursive definition of S.

Problem 2: Recall that a set M was called *countable* if there exists an injective function $f : M \to \mathbb{N}$. (For infinite M this is equivalent to the existence of a bijective function from M to $\mathbb{N}$.)

a) Show that the set $\mathbb{N}^2 := \mathbb{N} \times \mathbb{N}$ is countable.

b) Show that the rational numbers $\mathbb{Q}$ are countable.

c) Show that for any finite alphabet Σ the set Σ^* is countable. What's about countability of a subset of Σ^*?

d) Show that the real numbers are not countable.

Problem 3: For every of the following propositions write down its negation. Then decide whether the proposition or its negation is true. In all cases, Σ denotes a finite alphabet.

a) $\forall n \in \mathbb{N} \ \forall w \in \Sigma^* \ |w| \leq n$

b) $\exists w_1 \in \Sigma^* \ \forall w_2 \in \Sigma^* \ |w_1| \geq |w_2|$

c) $\forall f : \mathbb{N} \to \Sigma^* \ \exists w \in \Sigma^* \ f(1) \neq w$

Problem 4:

a) Let Σ_1 and Σ_2 be finite alphabets with cardinalities at least 2. Show that there is an injective function $\phi : \Sigma_2^* \to \Sigma_1^*$ such that $\forall w \in \Sigma_2^* \ |\phi(w)| \leq C \cdot |w|$ where $C = \lceil \log_{|\Sigma_1|} |\Sigma_2| \rceil$.

b) Show by a simple counting argument that if $|\Sigma_1| = 1$ then for any injective function ϕ as in a) there is an infinite sequence $\{w_i\}_{i \in \mathbb{N}}, w_i \in \Sigma_2^*$ such that $|\phi(w_i)| \geq |\Sigma_2|^{|w_i|}$.