

DM517 – Fall 2011 – Weekly Note 8

Final Pensum

- Michael Sipser, Introduction to the theory of computation, 2nd ed. pages 1-202 and 210-214.
- Notes by J. Bang-Jensen on undecidability (available for the home page of the course. 5 pages)
- All weekly notes and information on these.

Correction to the notes on (un)decidability

The notes are a rewrite of notes based on a previous book where TMs did not have two special cases q_{accept} , q_{reject} but only one halting state h . With the new definition (the one in Sipser) it is IMPOSSIBLE for any TM to use all its states on any input. Hence with that definition the problem of deciding whether a given TM will visit all its states on a given string is trivially decidable, since the answer is always NO. But if we instead change the problem to asking whether M will visit all states different from q_{accept} , q_{reject} , then we get an undecidable problem as shown in the lecture on October 7 (see slides).

Lecture in week 41, 2011:

I gave a overview of the course and showed how to solve some exam problems concerning (un)decidability.

Exercises October 11 and 14, 2011:

- 5.22 on page 216.
- 5.30 and 5.33 on Page 217
- January 2009, Problem 6
- January 2003, Problem 5.
- January 2004, Problem 4.
- January 2002, Problem 6.
- June 1996, Problem 3 (a).
- June 1996, Problem 5. Hint: reduce the halting problem to L .

- January 1998, Problem 5.
- January 1999, Problem 4. Note that there is a symbol missing in the definition of the language:
 - $L = \{ \langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset \}$
 - $L^* = \{ \langle M_1, M_2, w \rangle \mid M_1 \text{ og } M_2 \text{ stopper begge på } w \}$

Important classification of languages:

It follows from the results on decidability and recognizability that for every language L over the alphabet of the universal Turing machine, exactly one of the following holds:

- both L and \bar{L} are decidable.
- both L and \bar{L} are undecidable.
- precisely one of L, \bar{L} is recognizable and the other is not recognizable.

Some hints for the exam:

- READ all the problems before you starting solving any of them.
- Be sure that you know how to use the pumping lemmas for regular languages and context-free languages. Many of you have made errors in this in your hand in solutions. Take a (at least one) look at the smartboard slides concerning these things again and make sure you understand what you can choose freely and what you cannot choose. YOU choose the string s for which you want to obtain a contradiction. YOU DO NOT choose the partition of s into xyz for regular PL or $uvxyz$ for context-free PL. SO you have to show that no matter how this partition is done according to the “rules” in Theorems 1.70 and 2.34, there is SOME i such that xy^iz , respectively uv^ixy^iz is not in L . The key point is that by choosing s appropriately you can control the set of possible partitions of s . Note also that your chosen string must always be at least the pumping length p for the language. If you chose a shorter string, you cannot conclude anything!
- Make sure you know exactly the definition of a Turing machine, how it works, what it does when it reaches its special states q_{accept}, q_{reject} etc.
- Familiarize yourself with the structure of proofs for undecidability by reading and understanding the 3 examples in the notes.
- Make sure you understand the closure properties of regular, context-free and decidable languages as well as properties when you intersect with a regular language (e.g. if L is a CFL and L' is regular, then $L \cap L'$ is a CFL).

- Read all weekly notes very carefully! AND bring them to the exam along with all other parts of the pensum.
- Always make sure that you answer the question you are asked and not another one (read carefully).
- Always give an explanation for what you do.
- If you cannot solve a question but have an idea, write it down so that I can see whether you should get some of the points for that idea.
- Remember: the exam problems are made so that everyone who has followed the course, worked hard on the problems during the quarter and who follows my hints above will be able to pass the course easily.

If you have questions before the exam

I am in France from October 10 to 30. If you have an important question and cannot get help from a fellow student or Lena, you may send me an email presenting what the problem is and then I will try to help you. But do not send an email everytime you get stuck! Work seriously with all the material and ask only the remaining serious questions (if any).