Institut for Matematik & Datalogi
Syddansk Universitet

November 26, 2010
JBJ

DM528: Combinatorics, Probability and Randomized Algorithms — Ugeseddel 4

Pizzamøde: information om valgfrie kurser for forår 2011 IMADA holder pizzamøde for alle studerende mandag 29/11 kl. 16.15 i U49. Mødet vil indeholde generel information om kandidat- og bachelorstudiet, samt orientering om planlagte valgfri kurser i næste semester. Til slut vil der vre gratis pizza, øl og sodavand til de fremmødte.

Handing in obligatory assignment 1 This must be handed in to Magnus no later than at the exercise class on December 1st.

If you wish to get a receipt for having handed in the assignment you may do any of of the following things:

- Prepare a receipt to be signed by the instructor. The receipt should contain the names of everybody in the group who made the report. Make a copy for the instructor also so he can see which receipts he has signed.
- Hand in the assignment via Blackboard as follows:
  - Choose DM528 fall 2009
  - Choose "Course tools" in the menu left.
  - Choose "Assignment hand in"
  - Fill out the formular (including uploading the report as a PDF) and press submit
  - Print your receipt (you will get one via email also)

Stuff covered in week 47

- Rosen 6.2 last part on the probabilistic method.
- Rosen 6.3-6.4.
- Notes on Weekly note 3 about the probabilistic method
- Rosen 7.1-7.2.
- Rosen 7.3 about divide and conquer algorithms and recurrence relations if left for self study. Most of it was covered in DM507 (Cormen sections 4.1-4.3). The proof of the master theorem is not part of the pensum but you must know how to use it.

Lecture Monday November 29, 2010:

- Rosen 7.5-7.6

Exercises Wednesday December 1, 2010:

- Section 6.4: 32, 41,42,43.
- Supplementary exercises: 20, 30. 32
- Old exam problems 2004.06.5 and 2004.06.6 page 5

Exercises Friday December 3, 2010:

- Section 7.1: 18, 24, 28, 36 (nickel = 5 cent, dime = 10 cent), 40
- Section 7.2: 4(a), (f), 6, 8, 18 (hint: first show that r=2 is a root of multiplicity 3 in the characteristic equation), 24, 26(a),(b),(c)
- Section 7.3: 14.
- 2004.13.6 number 6 page 10 in notes with old exam problems.

Notes on indicator random variables

Let S be a sample space and let A be an event in S ($A \subseteq S$). Let X_A be the random variable which takes the value 1 when A occurs and 0 when A does not occour. That is,

$$X_A(s) = \begin{cases} 1 & \text{If } s \in A \\ 0 & \text{If } s \notin A \end{cases} \tag{1}$$

We say that X_A is the **indicator variable** for the event A. When A is clear from the context we drop the subscript A.

Theorem 1 *Let A be an event in a sample space S and let X be the indicator variable for A. Suppose the probability that A occurs is p ($p(A) = p$). Then expected value and variance of X is given by $E(X) = p$ and $V(X) = p(1-p)$.*

Proof:

$$\begin{aligned} E(X) &= 0 \cdot p(X=0) + 1 \cdot p(X=1) \\ &= 0 + 1 \cdot p \\ &= p. \end{aligned}$$

For the variance we want to use the formula $V(X) = E(X^2) - (E(X))^2$. This is easy to apply here since every indicator variable X satisfies that $X(s) = X(s)^2$ for all $s \in S$. Now we get

$$\begin{aligned} V(X) &= E(X^2) + (E(X))^2 \\ &= E(X) - (E(X))^2 \\ &= E(X)(1 - E(X)) \\ &= p(1-p). \end{aligned}$$

$\diamond$

Why are indicator variables useful? Because they allow us to simplify several proofs and to analyze probability of certain events with much less effort. Here is an example and there will be many more in the following weeks.

Theorem 2 *Consider n independent Bernoulli experiments each of which have probability of success equal to p. Let X be the random variable the denotes the number of successes in these n trials. Then $E(X) = np$ and $V(X) = np(1-p)$.*

Proof: Let X_i, $i = 1, 2, \ldots, n$ be the indicator random variable for the event that the ith trial (experiment) is a success (so $p(X_i = 1) = p$ for all $i = 1, 2, \ldots, n$). Then $X = \sum_{i=1}^{n} X_i$ so by linearity of expected values we get $E(X) = \sum_{i=1}^{n} E(X_i) = \sum_{i=1}^{n} p = np$. Furthermore, as X_i and X_j are independent random variables (the experiments are independent) for all distinct i, j we have $V(X) = \sum_{i=1}^{n} V(X_i) = \sum_{i=1}^{n} p(1-p) = np(1-p)$. $\diamond$