Institut for Matematik & Datalogi
Syddansk Universitet

December 10, 2010
JBJ

DM528: Combinatorics, Probability and Randomized Algorithms — Ugeseddel 6

Lecture Monday December 13, 2010:

- Cormen Algorithms book (the one from DM507) Section 5.1-5.3
- Kleinberg and Tardós: Sections 13.10-13.11

Lecture Friday December 17, 2010:

- The rest of the stuff in Kleinberg and Tardós: Sections 13.10-13.11.
- Rosen Section 7.4.

Exercises Wednesday December 15, 2010:

- Cormen Section 5.2: 5.2-1, 5.2-2
- Cormen Section 5.3: 5.3-4, 5.3-5
- From Kleinberg and Tardós but not in the notes:
  - Problem 10: Consider a simple online auction system working as follows. There are n bidding agents; agent i has a bid b_i which is a positive natural number. Assume that all bids are distinct. The bidding agents appear in an order chosen uniformly at random, each proposes its bid b_i in turn and at all times the system maintains a variable b^* equal to the highest bid seen so far (starting with $b^* = 0$ initially). What is the expected number of times b^* is updated when this process is executed, as a function of the parameters in the problem?

    Example Suppose $b_1 = 20, b_2 = 25$ and $b_3 = 10$ and bidders arrive in the order $1, 3, 2$. Then b^* is updated for 1 and 2 but not for 3.
  - Modified problem 2 from KT: Consider a voting situation with two candidates D and R. There are 100000 people who vote and 80000 of these are determined to vote for D, while the other 20000 are determined to vote for R. However, due to confusion from the way the ballot (valgseddel) is made, each voter independently and with probability $\frac{1}{100}$ votes for the wrong candidate, that is, the one (s)he didn't intend to vote for.

    Let X denote the random variable equal to the number of votes received by the candidate D when the voting is conducted with this process of error. Determine the expected value of X and explain your derivation of this value.
  - Modified problem 1 from KT: A **3-colouring** of a graph $G = (V, E)$ is a mapping $f : V \to \{1, 2, 3\}$. We say that G is **3-colorable** if we can choose f so that for every edge $uv \in E$ we have $f(u) \neq f(v)$. In this case we call f a **proper 3-colouring** of G. Not all graphs are 3-colourable (in fact as you will see in DM508, deciding whether a graph is 3-colorable is an NP-hard (very difficult) problem). Let f' be any mapping of V to $\{1, 2, 3\}$ we say that the edge uv is **satisfied** by f' if $f'(u) \neq f'(v)$. Hence a proper 3-colouring of G is

one which satisfies all edges of G. Let $c^*(G)$ be the maximum number of edges of G which can be satisfied by a 3-colouring f of G. Describe a polynomial algorithm which produces a 3-colouring which satisfies at least $\frac{2}{3}c^*(G)$ edges. Your algorithm can be deterministic or randomized; in this case the expected number of satisfied edges should be at least $\frac{2}{3}c^*(G)$.

- Cormen Problem 2 page 118 in the second edition and page 143 in the third edition.