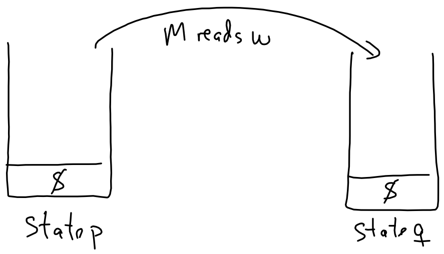
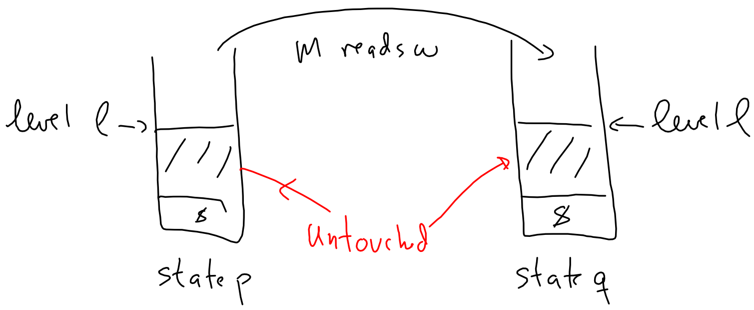
Idea in proof of Lemma 2.27:

For all choices of states $p,q \in Q(M)$ the grammar G will contain a variable A_{pq}

A_{pq} will generate all strings $w \in \Sigma^*$ s.t.

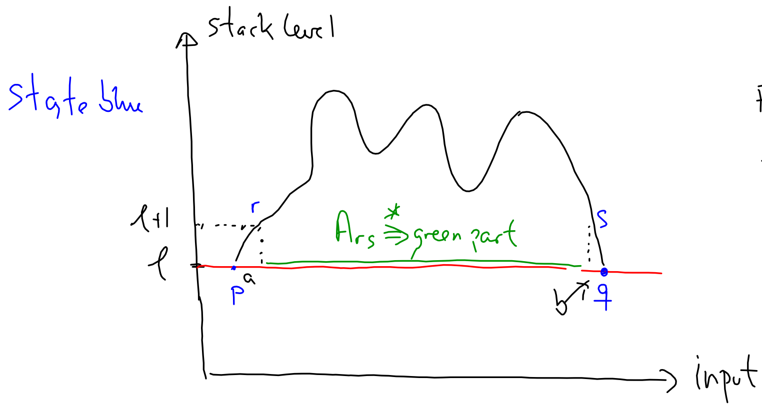


note that this is the same as:



2 posibilities while M eats w and goes from state p with stack level ℓ to state q and level ℓ never going below level ℓ:

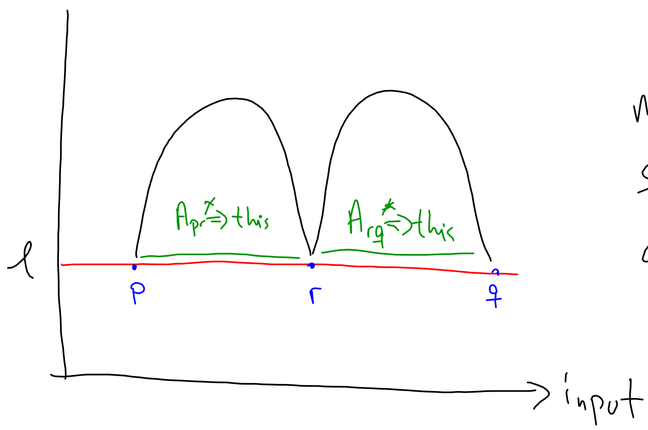
(a) M's stack is only at level ℓ initially and after all of w is read:



First step of M is to process some $a \in \Sigma_\varepsilon$ and PUSH some $\alpha \in \Gamma$

α stays on stack until the last step where M goes from state s to q while eating a $b \in \Sigma_\varepsilon$ from input and popping α

b) after reading a part of w and not all of w M is again at stack level ℓ



M starts again by pushing some $\alpha \in \Gamma$, but this time α is popped again before we reach the state just before q

Definition of G:

$V = \{ A_{pq} \mid p,q \in Q \}$

$S = A_{q_0 q_{accept}}$

Rules:

- $\forall p,q,r,s \in Q$ $t \in \Gamma$ and $a,b \in \Sigma_\varepsilon$:

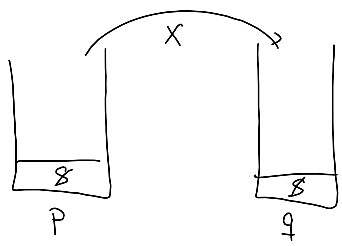
  If $p \xrightarrow{a,\varepsilon \to t} r$ $\wedge$ $s \xrightarrow{b,t \to \varepsilon} q$ are transitions of M

  then add $A_{pq} \to a A_{rs} b$ to R

- $\forall p,q,r \in Q$ add $A_{pq} \to A_{pr} A_{rq}$ to R

- $\forall p \in Q$ add $A_{pp} \to \varepsilon$ to R

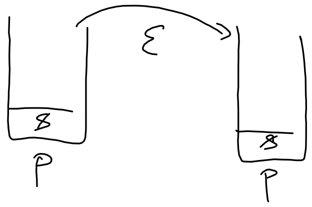
Claim 2.30 $A_{pq} \overset{*}{\Rightarrow} x \Rightarrow$



<u>proof</u> induction over # steps in derivation $A_{pq} \overset{*}{\Rightarrow} x$

<u>1 step:</u> then $p=q$ and $A_{pp} \to \varepsilon$ $(x \in \Sigma^*)$

clearly



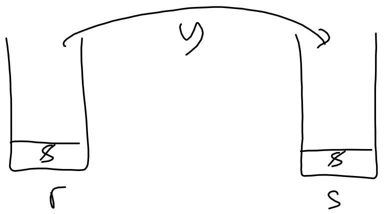
M does nothing.

$\boxed{\text{assume ok if } \le k \text{ steps in derivation}}$

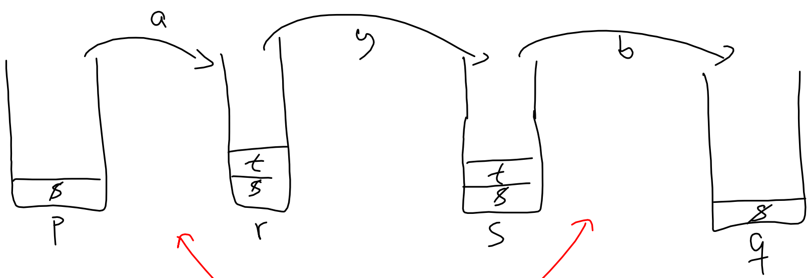
<u>k+1 steps:</u> look at first rule applied in derivation

- If $A_{pq} \to a A_{rs} b$ then $x = ayb$ and $A_{rs} \overset{*}{\Rightarrow} y$

so by induction



Thus

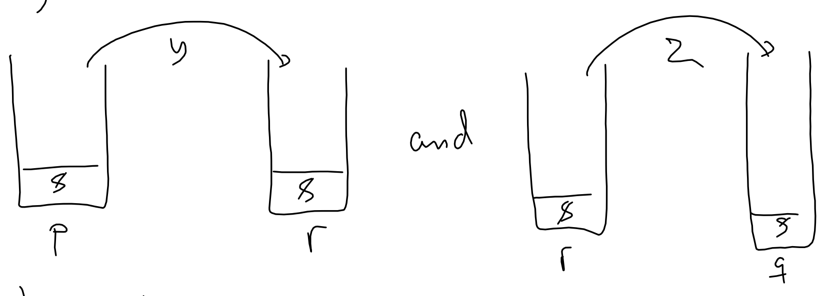


follow from the fact that we added $A_{pq} \to a A_{rs} b$ to R

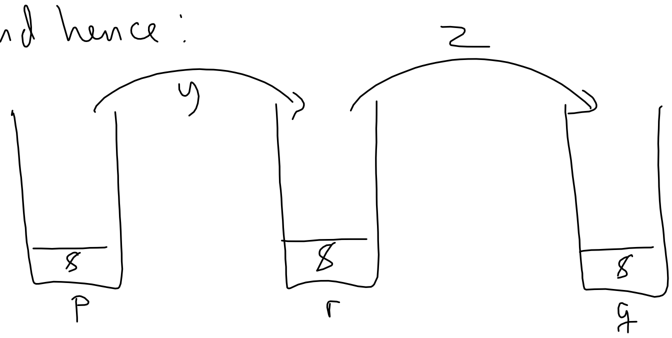
- If $A_{pq} \to A_{pr}A_{rq}$ then

  $x = yz$ and $A_{pr} \overset{*}{\Rightarrow} y$, $A_{rq} \overset{*}{\Rightarrow} z$

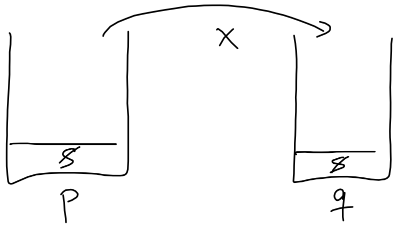
  so by induction:



and hence:



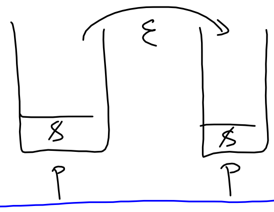
Claim 2.31



$\Rightarrow A_{pq} \overset{*}{\Rightarrow} x$

proof by induction on # steps in M's computation

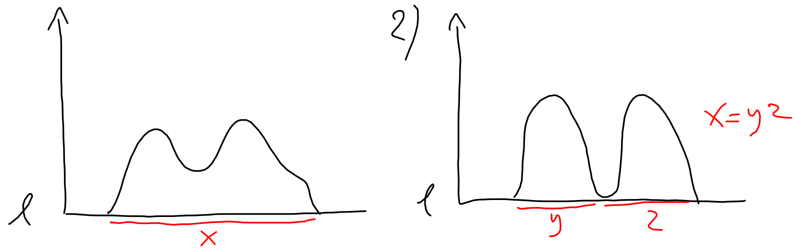
0 steps M cannot read anything in 0 steps
so $x = \varepsilon$ and $p = q$



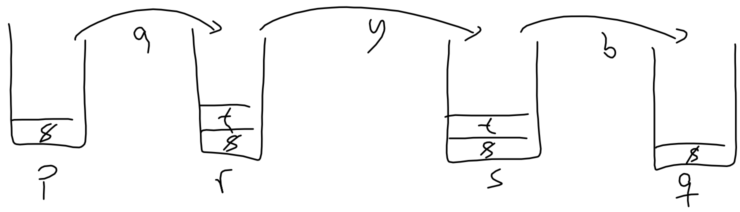
and $A_{pp} \to \varepsilon$

$\boxed{\text{assume ok if } M \text{ takes} \le k \text{ steps}}$

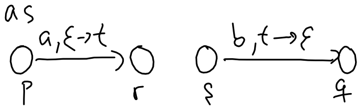
Two cases 1)



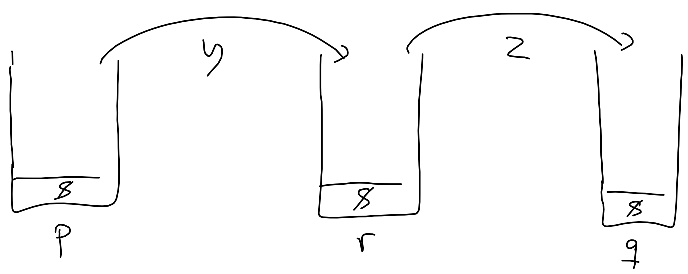
1) M's computation looks like:



so $x = ayb$ and
$A_{pq} \to aA_{rs}b \in R$
as



2) M's computation is of the form



by induction
$A_{pr} \overset{*}{\Rightarrow} y$
$A_{rq} \overset{*}{\Rightarrow} z$

So $A_{pq} \Rightarrow A_{pr}A_{rq} \overset{*}{\Rightarrow} yz = x$