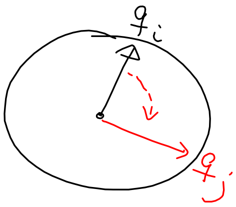
Turing machines

"reading" head

a	a	a	b	b	b	$\cancel{c}$ X	c	c	

infinite

initial state q_0
accept state q_{acc}
reject ... q_{reject}



$$\delta(q_i, c) = (q_j, X, L)$$

$$L = \{a^n b^n c^n \mid n \geq 0\}$$ not CFL

a	a	a	b	b	b	c	c	c	⊔	⊔	---- ⊔

q_0

config

We saw an 'a' ⇒ look for b $A\,q\,abbcc$

$\underline{a}abbcc \to A\underline{a}bbcc \to Aa\underline{b}bcc$

$\to AaB\underline{b}cc \to AaBb\underline{c}c \to AaBb\underline{C}c$

$\to \cdots \to \underline{A}aBbCc \to A\underline{a}BbCc \to AA\underline{B}bCc \to AAB\underline{b}Cc$

$\to AABB\underline{C}c \to AABBC\underline{c} \to AABB\underline{C}C$

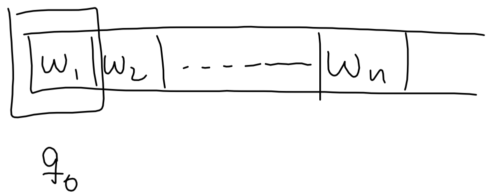
$\cdots \to A\underline{A}BBCC \to AABBCC\underline{⊔}$

config: $AABBCC\,q_{acc}$ accept!

Def 3.3 $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

$\sqcup \notin \Sigma \qquad \Sigma \subseteq \Gamma$

initially



Configuration : $uqv \qquad u, v \in \Gamma$

$$\Updownarrow$$

$$u_1 \cdots u_r \underset{q}{\underline{v_1}} v_2 \cdots v_s$$

Configuration C_1 yields C_2

$\Updownarrow$ M can go from C_1 to C_2 in one step.

$u a q_i b v$ yields $u q_j a c v$

$\Updownarrow$

$\delta(q_i, b) = (q_j, c, L)$

NB: $q_i b v \rightarrow q_j c v$ if $\delta(q_i, b) = (q_j, c, L)$

start config $q_0 w$

accepting conf $u q_{acc} v$

rejecting conf $u' q_{reject} v'$

$L(M) = \{ w \mid q_0 w$ yields $u q_{acc} v$ for some $u, v \in \Gamma^* \}$

in ~~one or~~ several steps

$$\boxed{\begin{array}{l} C_1 = q_0 w \\ C_i \rightarrow C_{i+1} \\ C_n = u q_{acc} v \end{array}}$$

Def 3.5 $L \in \Sigma^*$ is Turing recognizable

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ " recursively enumerable

$\Updownarrow$ def

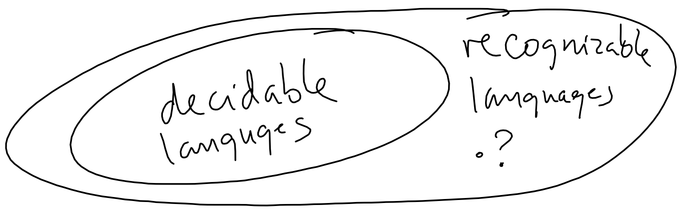
$L = L(M)$ for some Turing machine M

Three possible outcomes when M is started on w

- M accepts w $\quad\quad q_0 w \rightsquigarrow u\, q_{acc} v$
- M rejects w
- M 'loops' forever.

A decider is a TM that always stops

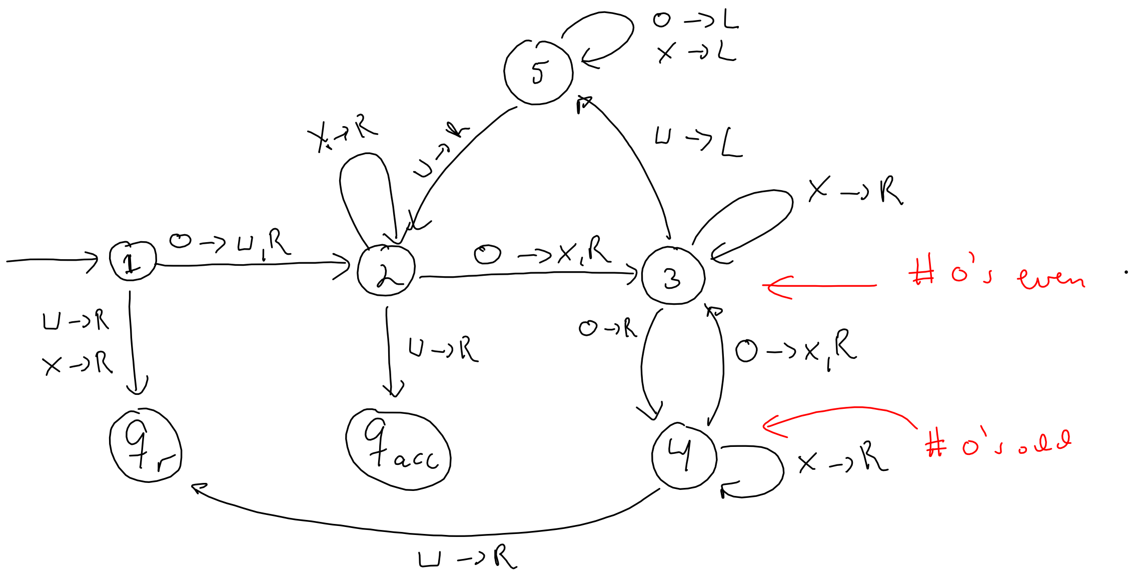
L is decidable $\overset{def}{\iff}$ $L = L(M)$ for some decider TM.



Ex 3.7 Decider for $A = \{0^{2^n} \mid n \geq 0\}$

1. Cross out every second '0' while moving right
2. If there was only one '0' accept
3. If ------- odd # of '0's reject
4. return to left hand end of tape
5. Goto 1.

$\underline{0}000 \dashrightarrow \acute{x}0x0\underline{\sqcup}$



Notation: 0 → ⊔, R : when reading '0' write '⊔' and move right

0 → R when reading '0' move right without changing tape (print '0')

Usefull TM's not in book

$q_0 w_1 w_2 \cdots w_n \longrightarrow \triangleright q_0' w_1 w_2 \cdots w_n$

Right shifting $\leftarrow R_{shift}$

$\underline{w_1} w_2 \cdots w_n \dashrightarrow \mathring{w}_1 \cdots w_n \underline{\sqcup}$

$\mathring{w}_1 w_2 \cdots \underline{w_n} w_n \rightarrow \mathring{w}_1 \cdots \underline{w_{n-1}} \sqcup w_n$

$\mathring{w}_1 \cdots w_{n-1} w_{n-1} w_n \rightarrow \mathring{w}_1 \cdots \underline{w_{n-1}} w_{n-1} w_n$

$\dashrightarrow \underline{\mathring{w}_1} w_1 \cdots w_n \rightarrow \triangleright w_1 \cdots w_n$

Notes

1. R_{shift} never touched left hand side of tape

2. Could also have done something like

   $\underline{w_1} w_2 \cdots w_n \longrightarrow \sqcup w_1 w_2 \cdots w_n$

   and $w q w' \longrightarrow \underline{w} \sqcup w'$

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