<u>Claim</u> If L is regular, then L is context-free

P: let M be DFA s.t $L(M)=L$

$Q=\{q_0, q_1, \dots q_k\}$ states of M

Variables in G:

$X_0, X_1, \dots, X_k$

Rules:

$q_i \xrightarrow{a} q_j \Rightarrow$ add $X_i \to aX_j$ to R.

Finally add $X_e \to \varepsilon$ if $q_e \in F = F(M)$

X_0 is starting symbol of G.

suppose $w \in L$: $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \cdots \quad q_e$

$X_0 \Rightarrow aX_1 \Rightarrow abX_2 \cdots \Rightarrow wX_e \Rightarrow w$

so $w \in L(G)$

Conversely: If $w' \in L(G)$

then $X_0 \overset{*}{\Rightarrow} w'$ must be of the form

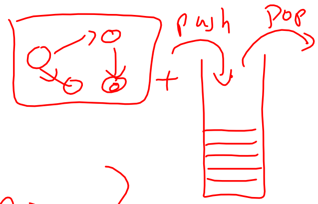
$X_0 \Rightarrow a'X_i \Rightarrow \cdots\cdots \Rightarrow w'X_p \Rightarrow w'$

$p \in F(M)$

<u>Fact</u> : If L is a context-free language over Σ where $|\Sigma| = 1$ then L is also regular.

<u>Corollary</u> when $|\Sigma| = 1$ L is regular
$\Updownarrow$
L is context-free

Stack automatons



Idea for $L=\{a^nb^n \mid n\geq 0\}$

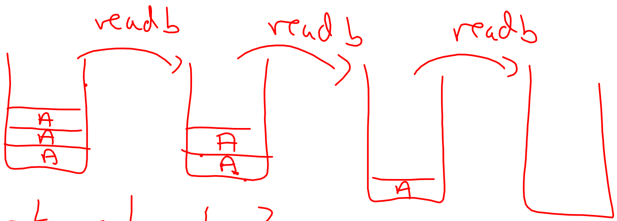
while reading a's: put 'A' on stack

when seeing a b : pop 'A' from stack

while reading b's : pop 'A's

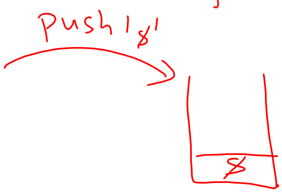
if stack $= \emptyset$ and w read accept

aaabbb →

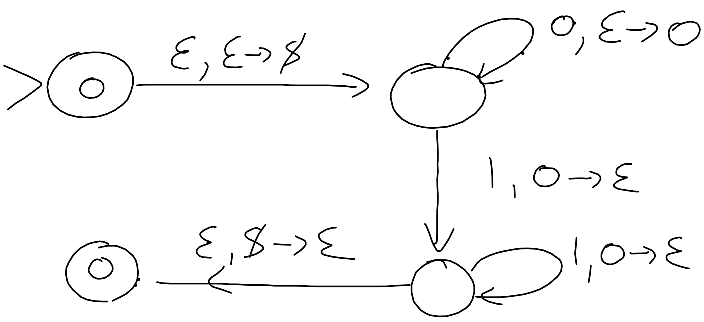


How to detect empty stack?

step 1 put special symbol '\$' on stack



PDA for $\{0^n1^n \mid n\geq 0\}$:



PDA for $L = \{a^i b^j c^k \mid i,j,k \geq 0,\ i=j \text{ or } i=k\}$

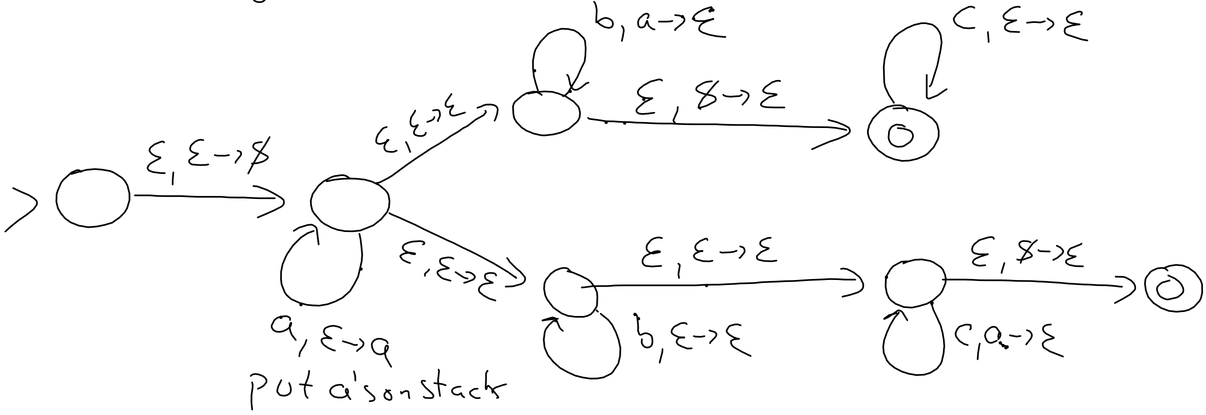
<u>NB</u> L is not regular:

$L \cap a^*b^* = \{a^n b^n \mid n \geq 0\}$ not regular

so L is not regular since regular languages are closed under intersection.

How to make PDA ^{M} for L?

Let M guess whether we have $i=j$ or $i=k$ above

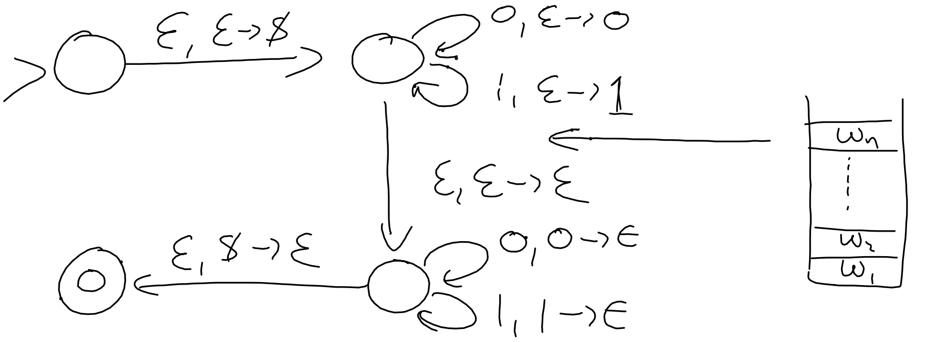


! <u>does it work?</u>

1. $a^3b^3c^2 \in L$ ✓ $\quad a^3b^2c^2 \notin L$ ✓
2. $a^3bc^3 \in L$ ✓ $\quad a^3bc^2 \notin L$ ✓
3. $\varepsilon \in L$ ✓ $\quad a^2b^3c^4 \notin L$

Ex 2.18 $\quad L = \{ w w^R \mid w \in \{0,1\}^* \}$

$w = w_1 w_2 \cdots w_n \qquad w^R = w_n w_{n-1} \cdots w_2 w_1$



Not all languages are context-free!!

e.g. $L = \{a^n b^n c^n \mid n \geq 0\}$ is not CFL

How to prove this??

pumping "some kind of"

Thm 2.34

$\forall$ CFL L $\exists$ $p \in \mathbb{N}$ s.t every $w \in L$ s.t

$|w| \geq p$ then $\exists$ $uvxyz \in \Sigma^*$ s.t.

(0) $w = uvxyz$

(1) $uv^i x y^i z \in L$ $\forall i \geq 0$

(2) $|vy| > 0$

(3) $|vxy| \leq p$

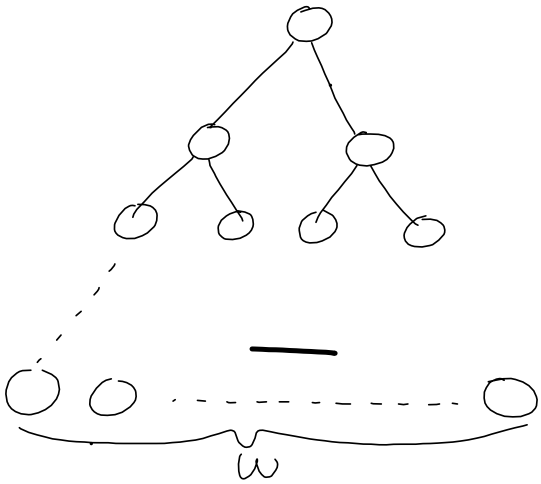
P: let G be a chomsky NF grammar s.t

$L = L(G)$ and let $p = 2^{|V|+1}$

where V is the set of variables of G

Suppose $|w| \geq p$

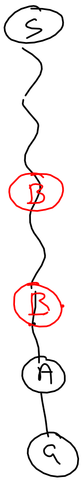
look at a parsetree for w, $|w| \geq 2^{|V|+1}$



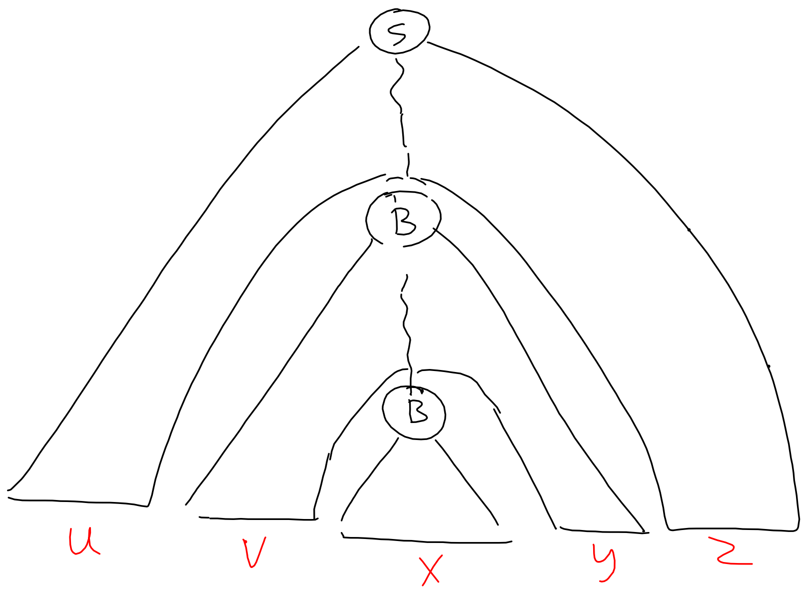
$\geq 2^{|V|+1}$ leaves

so height is at least $|V|+2$

(last step is of the form $A_i \to a$



$\leftarrow$ length $|V|+2$



$w = uvxyz$ (0| ✓

$uv^ixy^iz \in L$

$|vy| > 0$

$|vxy| \leq p$