Non Deterministic TMs NDTM

Ex Check whether n not a prime

1. Guess $n_1, n_2 \geq 2$
2. check $n = n_1 * n_2$
3. If yes answer not prime

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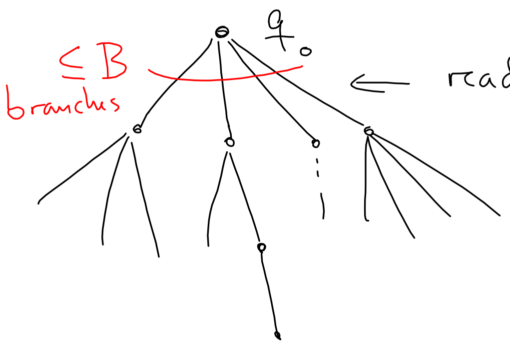
no of possible transions on a symbol $b \in \Gamma$ from state q

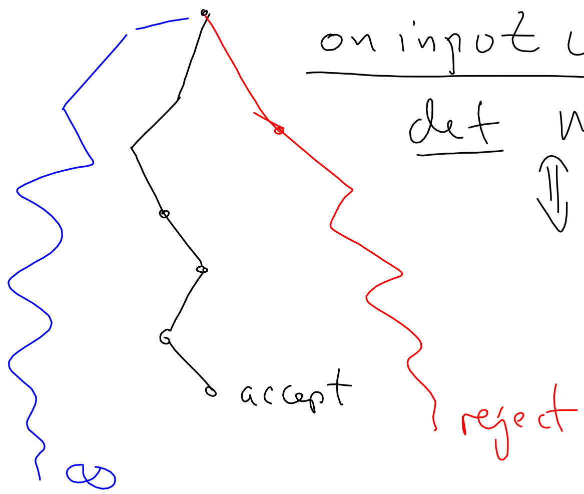
$\delta(q,b)$ is at most $\underbrace{|Q| \cdot |\Gamma| \cdot 3}_{B}$ ← $\{R, L, S\}$

M's computation on w can be viewed as a tree



← read symbol under head

Could have following possibilities



on input w

def M accepts w

$\Updownarrow$ $\exists$ accepting path

M accept L $\quad (L = L(M))$

if $\forall w \in L$ $\exists$ accepting path for M on w

and $\forall w \notin L$ $\nexists$ - - - - - -

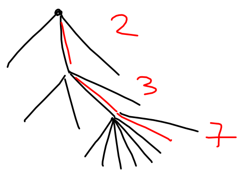
How to define deciding for NDTM

1. $\exists\ K = K(M, w) \in \mathbb{Z}$
   s.t no path from computation has length more than K
2. There is at least one accepting path for M on w

Thm 3.16 $\forall$ NDTM M
$\exists$ DTM D
s.t $L(M) = L(D)$ and
D decides L iff M decides L

proof idea simulate M on w
using strings of numbers
in base $B+1$, e.g. 237 $(B \geq 7)$

no zero's



Conclusion: If ~~X~~ we can implement
the step above then we will
have $L(M) = L(D)$

How do we simulate using numbers?
May assume that $\delta(q,a)$ always has B possibilities
by copying one of the transitions enough
times

How generate numbers like 237
and in which order?
Lexicographic ordering on strings of the same length
237 < 239

also 237 < 1117 because shorter

simple way to keep track of numbers

0 0 # 0 0 0 # 0 0 0 0 0 0 0
2 3 7

D has 3 tapes: # w untouched
M's calculation on w
numbers base (B+1)

How long does it take to Simulate a NDTM with at most B choices per step?

suppose M uses r steps in a calculation

then D uses at most

$$B + B^2 + \dots + B^r \le B^{r+1} \text{ steps}$$

so we use exponential time in r !!

Q: can we do it in pol #steps $(O(r^c))$
c const

open 10^6 \$ if solved

$(P \stackrel{?}{=} NP)$

The Universal Turing machine

Universal alphabet $A^* = \{a_1, a_2, a_3, \ldots\ldots\}$ ∞

— " — state set $Q^* = \{q_1, q_2, \ldots\ldots\}$ ∞

Given TM M with states Q and symbols Γ

s.t $r = |Q|$ and $t = |\Gamma|$

then we may rename Q to $\{q_1, q_2, \ldots, q_r\}$ (first r states of Q^*) and

Γ to $\{a_1, a_2, \ldots a_t\}$ (first t symbols of Γ^*

Conclusion: Every TM M has an equivalent TM M' with $Q \subseteq Q^*$ ("prefix") $\Gamma \subseteq \Gamma^*$ "prefix")

and $q_i \sim q_{i \text{ in binary}}$ $(q_7 \sim q_{111})$ do for a_j

Coding of TM M:

List of tuples of the kind $((q_i, a_j), (q_s, a_q, \overset{\{R,L,S\}}{\xi}))$

$\langle M \rangle = ((q_1, a_1), (q_{f}, a_7, R))((q_1, a_2)(\quad))\ldots\ldots ((q_r, a_t)(q_b, a_e, L))$

If $w = a_7 a_1 a_2 a_3$ then $\langle w \rangle = (a_7)(a_1)(a_2)(a_3)$

Universal TM U

Simulating a TM M on w

3 tapes:

initialy 1: $\langle w \rangle$

2: $\langle M \rangle$

3: q_1

one step of simulation

let q = state on tape 3

a = symbol on tape1 (under head of tape1)

Find entry for (q, a) on tape2 and perform changes on tapes 1 and 3

If $((q,a)(p,b,L))$

new symbol on tape1 is b

new state on tape3 is p

U will accept M, w

$\Updownarrow$

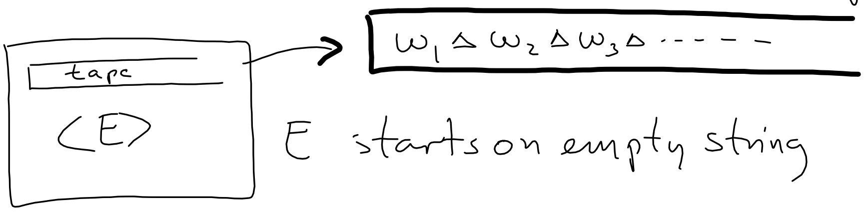
M accepts w

and a DTM M will halt on w ← stop.

iff U will halt on M, w

Enumerators

Turing machine with an output tape



E starts on empty string

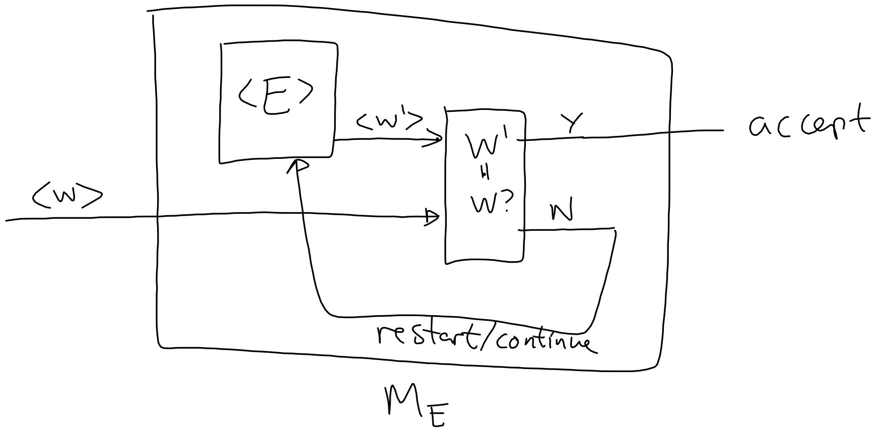
$L(E)$ = set strings printed by E (over ∞ time)

L is enumerable if $L = L(E)$ for some enumerator E

Theorem 3.21 L is enumerable ⟺ $L = L(M)$ for some DTM M

⟹:

Diagrams:



Clearly $L(M_E) \subseteq L(E)$

also $L(M_E) \supseteq L(E)$ because E will eventually print any string in $L(E)$

$\Leftarrow$ let $L = L(M)$ M DTM

let Σ be the alpabet for M

and order Σ^* as $s_1, s_2 \ldots\ldots\ldots s_k, s_{k+1} \ldots$

lexicographically

E_M: For $i = 1$ to ∞

simulate M for i steps on

$s_1, s_2 \ldots s_i$ **and** print s_j if M accepts

it in less than $i+1$ steps

$\# s_1 \# s_2 \# s_3 \# \ldots \# s_i \#$

If M accepts w in, say p steps and

$w = s_k$ then E will print in all steps $n \geq \max\{p, k\}$

so $w \in L(M) \Rightarrow w$ is printed (∞ many times)

and E only prints strings from $L(M)$

so $L(E) = L(M)$.