

Written exam

Computability (DM517)

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You are allowed to use the textbook, your personal notes, and a pocket calculator. The exam contains 6 pages and 5 problems, each on a single page. A complete solution consists of a solution for all 5 problems. How much a problem weights can be seen from the numbers given in brackets for each problem. In general you may refer to results from the book unless it is explicitly stated that you may not. The same holds for problems solved in the exercise sessions. Of course, if you cite such a result, it must be obvious that your claim really is a straightforward consequence of that result. It will not be accepted as an answer to refer to other books than the textbook. Recall that you have to explain all your claims!

Your answers must be in English or Danish.

1 Problem 1 (25%)

Let the language L_1 over alphabet $\Sigma = \{a, b\}$ be defined as

$$L_1 = \{abw : w \in \Sigma^* \text{ and } |w| \geq 2 \text{ and } (w_{|w|-1} = b \text{ or } w_{|w|} = b)\}$$

Question a:

Specify whether each of following strings is in L_1 or not.

- abb
- abbaa
- abaab
- abba

Question b:

Construct a regular expression for L_1 .

Question c:

Construct a DFA which accepts L_1 .

Question d:

Construct a NFA which has at most six states and accepts L_1 .

Question e:

Is the language $(L_1L_1) \cup L_1$ recognizable? Prove your answer.

2 Problem 2 (25%)

Let $L_2 = \{a^i b^j c^k : i \leq j + k\}$ and $L_3 = \{a^i b^j c^k : i > j + k\}$ be languages over the alphabet $\{a, b, c\}$.

Question a:

Prove that L_2 is not regular.

Question b:

Construct a context-free grammar for L_2 .

Question c:

Is $L_3 = \overline{L_2}$?

Question d:

Construct a PDA that accepts L_3 .

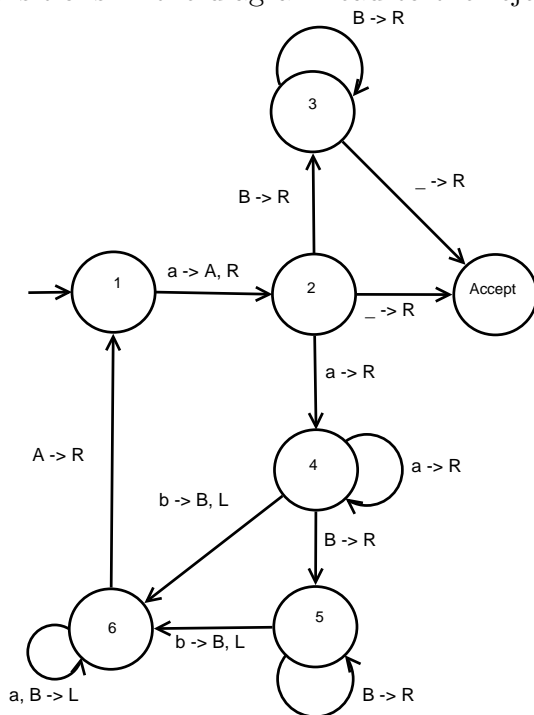
3 Problem 3 (10%)

Prove or disprove the following claims (a counterexample is sufficient to disprove a claim).

1. The language $A \cap B$ is regular if and only if A and B are finite.
2. If A and B are non-regular, then $A \cup B$ is non-regular.
3. If A is finite and B is context-free, then $A \cup B$ is context-free.

4 Problem 4 (15%)

Consider the Turing machine M in the figure below. M uses the alphabets $\Sigma = \{a, b\}$ and $\Gamma = \{a, b, A, B, _ \}$, where $_$ is the blank symbol. All missing transitions in the diagram lead to the reject state.



Question a:

Specify whether M accepts or rejects each of the following strings.

- a
- b
- aa
- $aabb$
- aab
- $aabbb$
- $aaabb$

Question b:

Describe the language decided by M .

5 Problem 5 (25%)

For each of the languages below, specify whether the language is decidable or not. For each language which is undecidable, prove that this is the case. For each language which is decidable, give a brief and informal argument that this is the case. Remember that $L(M)$ is the language recognized by Turing machine M , i.e. the set of strings on which M halts in its accept state.

1. Given the encoding of a Turing machine M , is $L(M)$ a nonregular language?
2. Given the encoding of a Turing machine M and a string w , does M have $|w|$ states?
3. Given the encoding of two Turing machines M_1 and M_2 , does there exist a language L such that $L \cap L(M_1) \subseteq L(M_2)$?
4. Given the encoding of a Turing machine M and a string w , does M take at most $|w|$ steps when given w as input?
5. Given the encoding of two Turing machines M_1 and M_2 and a string w , does M_1 take more steps than M_2 when they are given w as input?
6. Given the encoding of a Turing machine M and an integer k , is $|L(M)| = k$?