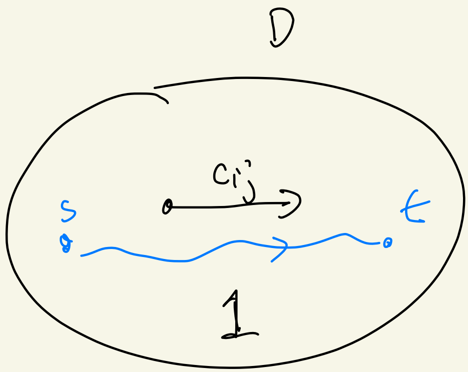
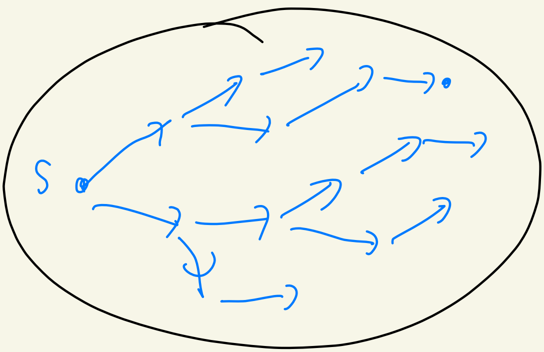
Ahuja 1.1

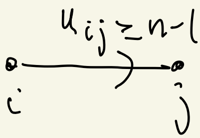
shortest path problem

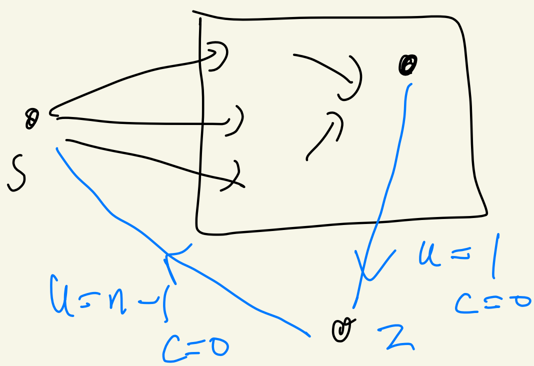




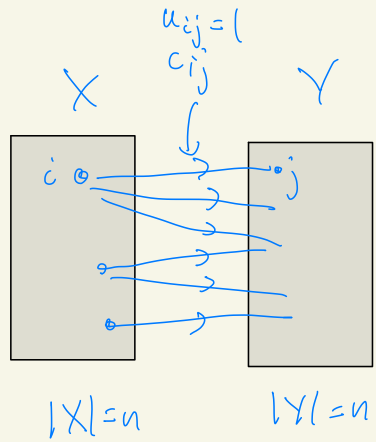
$$b_x(s) = n-1$$

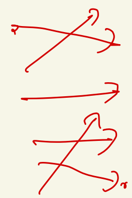
$$b_x(v) = -1 \quad \forall v \neq s$$





assignment

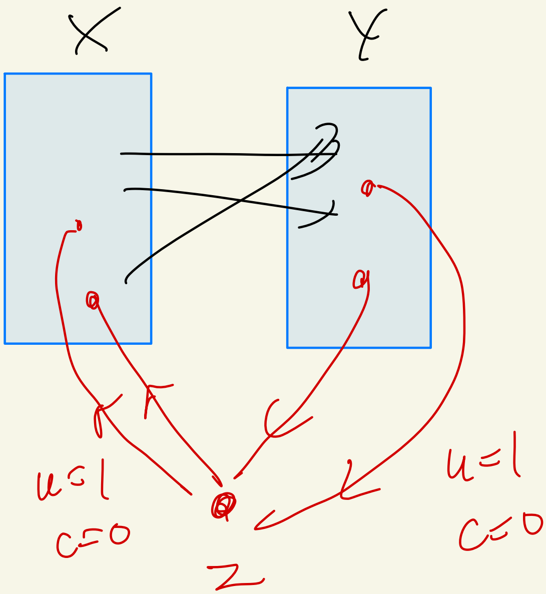




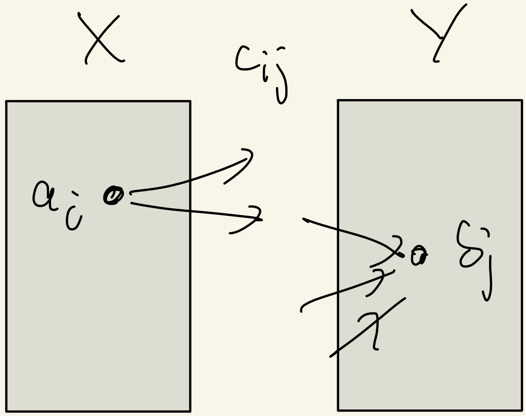
min sum of c_{ij}'s

$b_X(v) = 1$ if $v \in X$

$b_X(v) = -1$ if $v \in Y$

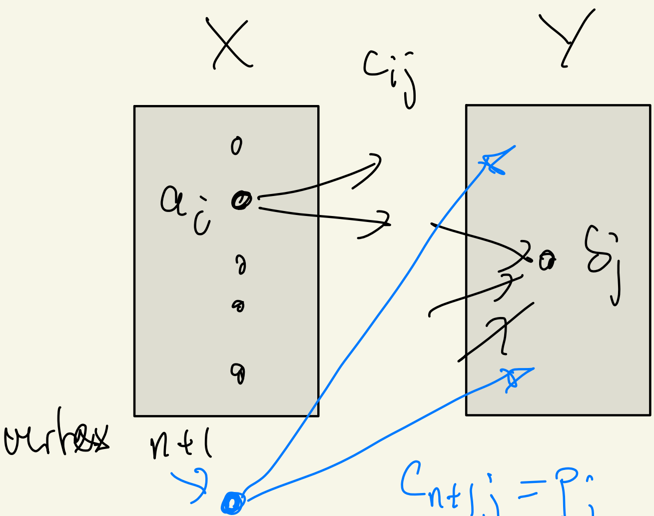


Ahuja 1.2



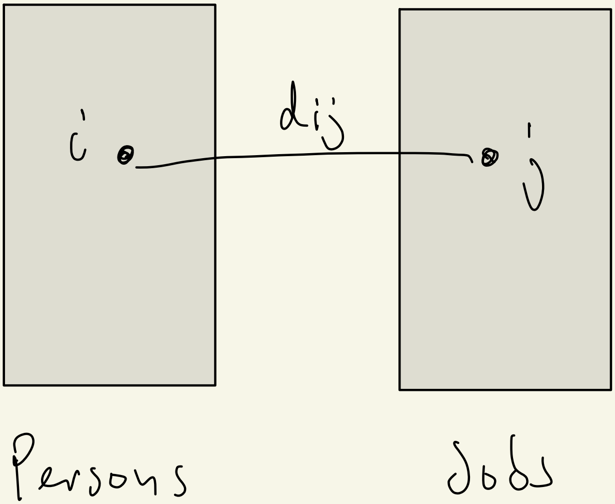
transportation problem $\sum_i a_i = \sum_j b_j$

suppose $\sum_j b_j > \sum_i a_i$



$a_{n+1} = \sum_j b_j - \sum_i a_i$

Ahuja 1.4



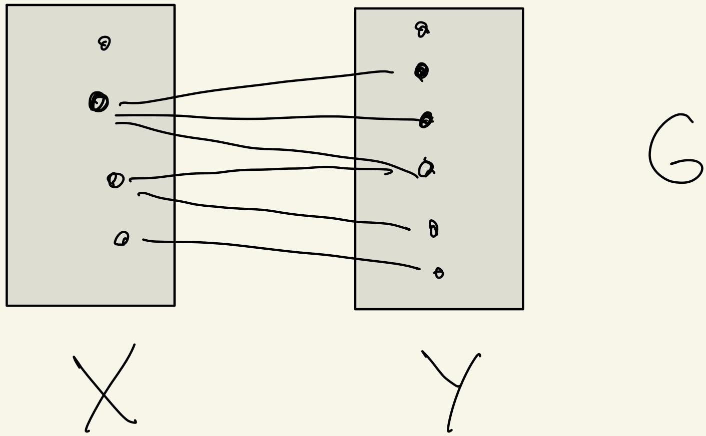
d_{ij} = utility of assigning person i to job j

want to maximize total utility

cost $c_{ij} = -d_{ij}$

Ahuja 1.5

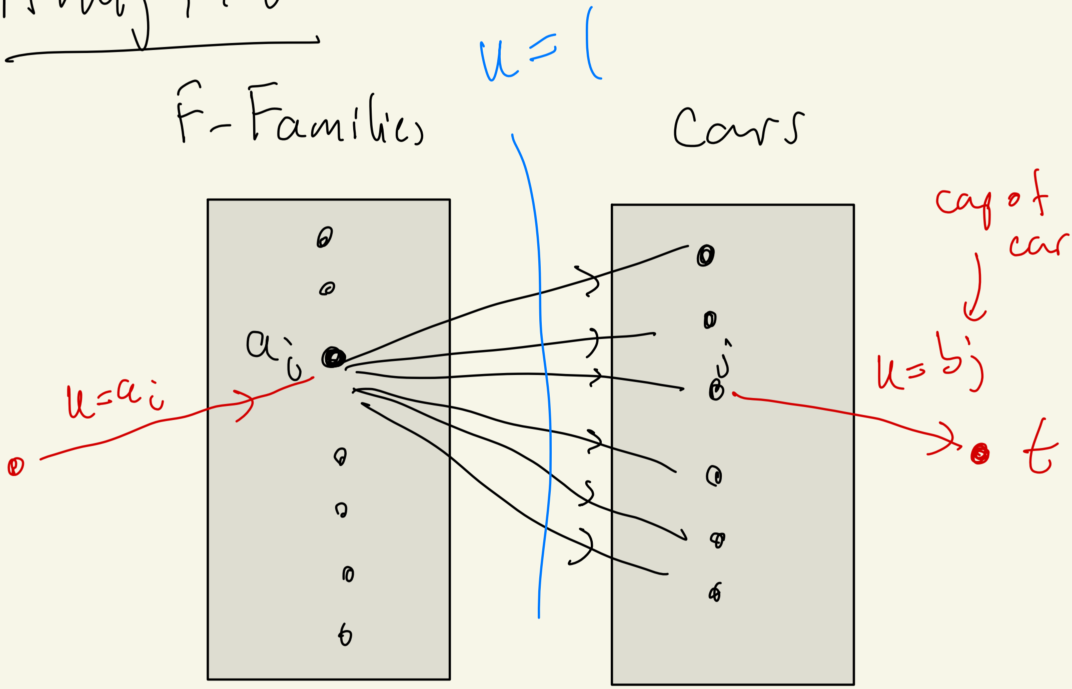
Dating problem



find max no of compatible x — y

= max matching in G

Ahuja 1.8



$$\sum_{i \in F} a_i$$

integer

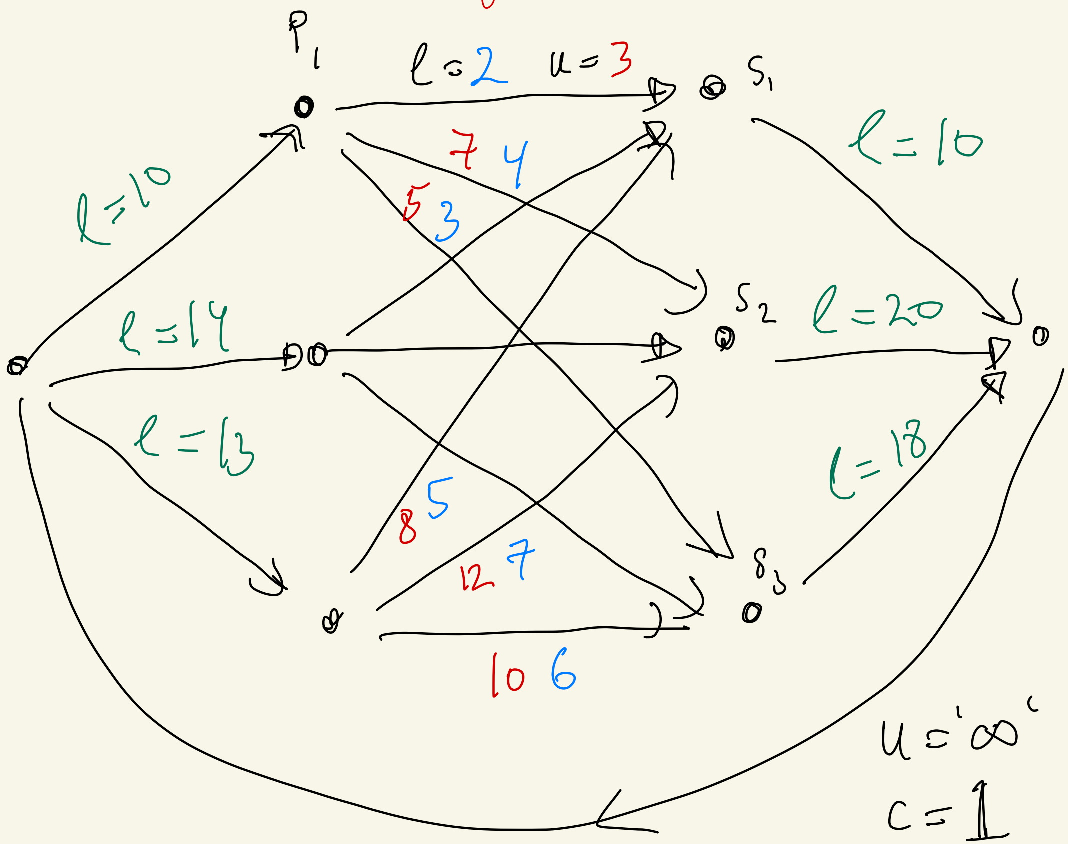
good distribution $\longleftrightarrow$ flow of value $\sum_{i \in F} a_i$

Ahuja 1.9

	S_1 ≥ 10	S_2 ≥ 20	S_3 ≥ 18
≥ 16 P_1	2 3	4 7	3 5
≥ 14 P_2	3 5	6 7	5 10
≥ 13 P_3	5 8	7 12	6 10

i = minimum # of cars

j = maximum # of cars



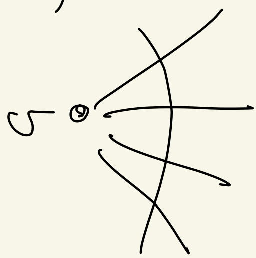
$u = \text{'}\infty\text{'}$

$c = 1$

Ahuja 2.12

$G = (V, E)$

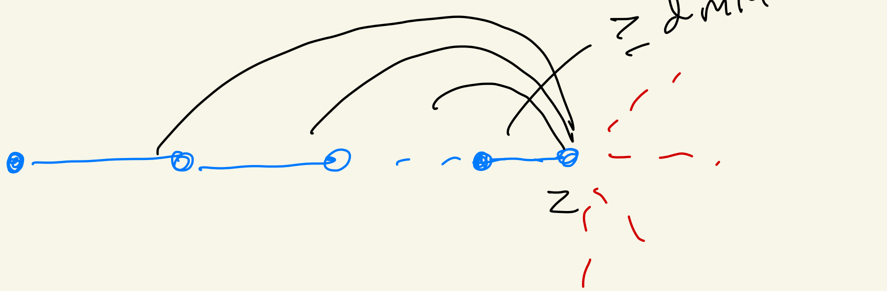
$d(v)$



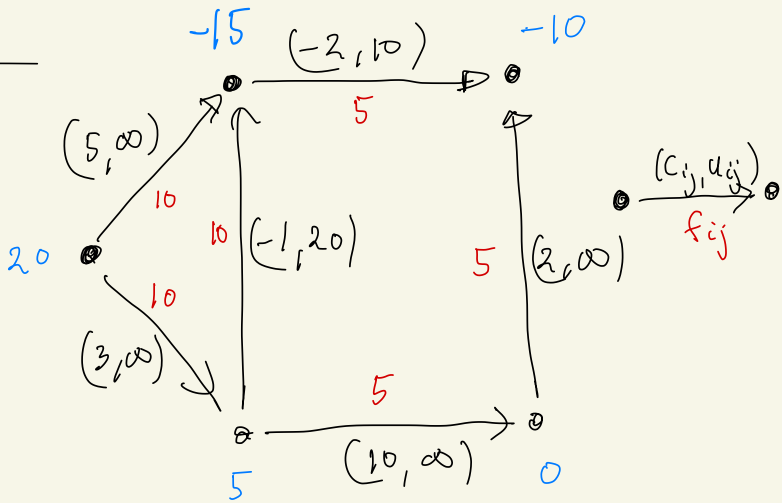
\# of edges incident to v

$d(v) \geq d_{min} \quad \forall v$

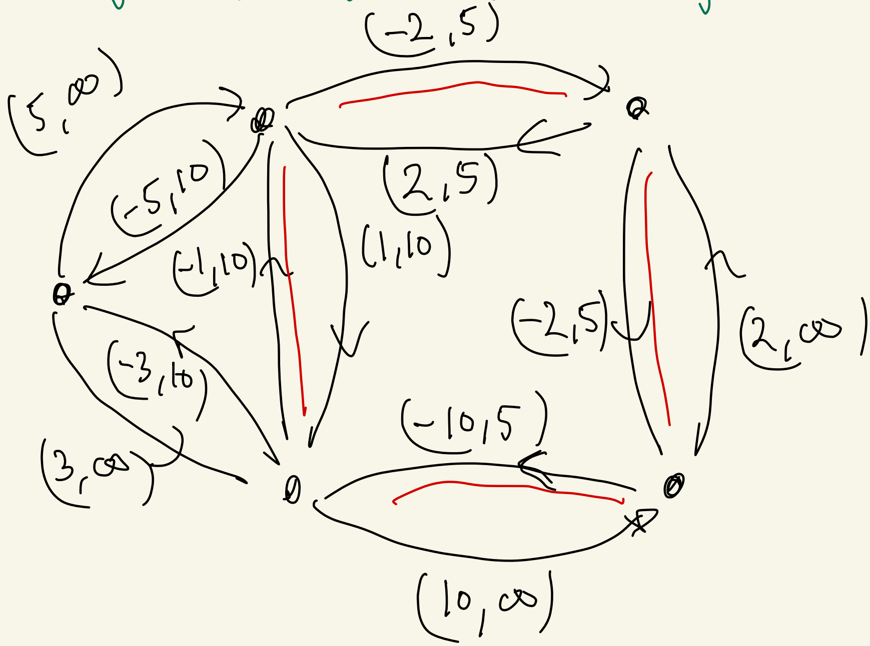
Show that G has a path with at least d_{min} edges



Ahuja 2.45



$r_{ij} = (u_{ij} - x_{ij}) + x_{ji}$ $\quad l_{ij} = 0 \;\forall ij$

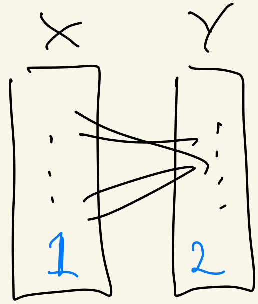


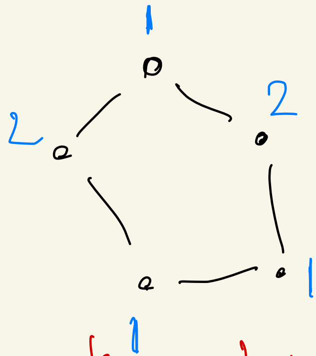
red cycle has cost −15
so flow is not min cost

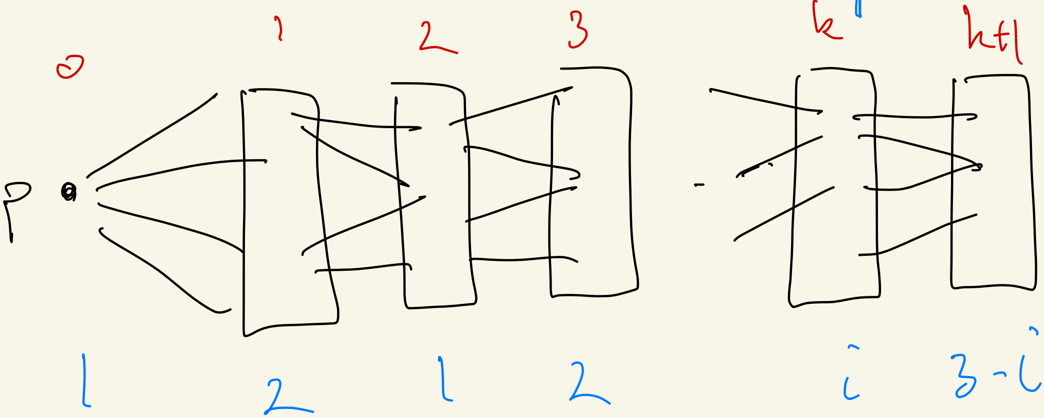
G is bipartite

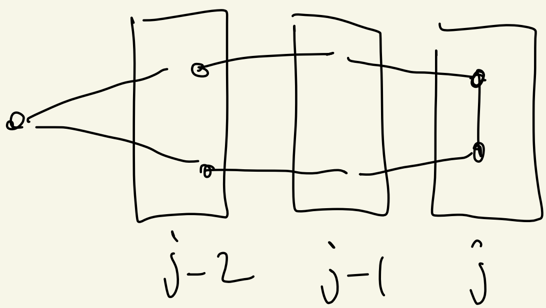
$\Updownarrow$

G has no cycle of odd length







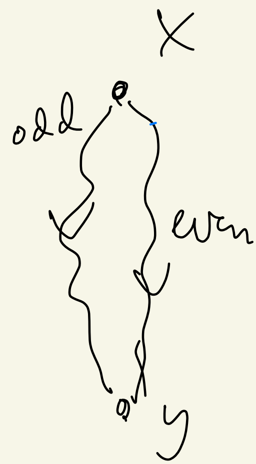


edge inside distance class $\Longleftrightarrow$ odd cycle

Prove that a strongly connected digraph is bipartite if and only if it has no directed cycle of odd length

$\Rightarrow$ ✓

$\Leftarrow$ fix vertex x



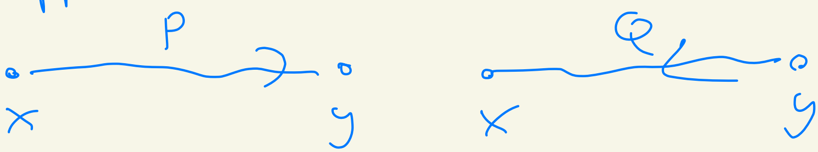
Claim: $\forall y \in V - x$

$$|A(P_1)| \equiv |A(P_2)| \mod 2$$

$\forall$ (x,y)-paths

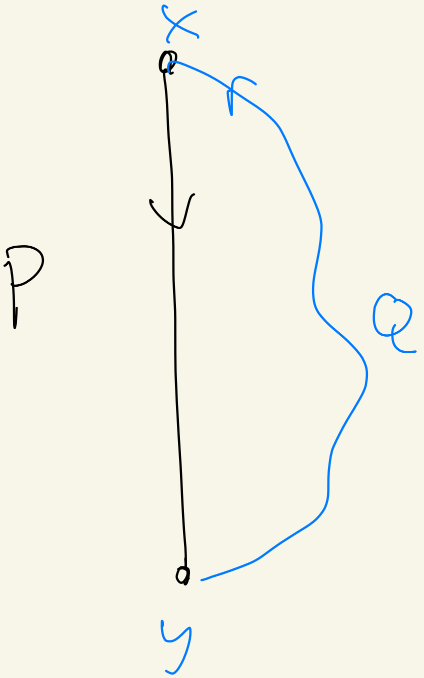
proof suppose not. Choose P and Q

s.t.



s.t. $|A(P)| \not\equiv |A(Q)| \mod 2$

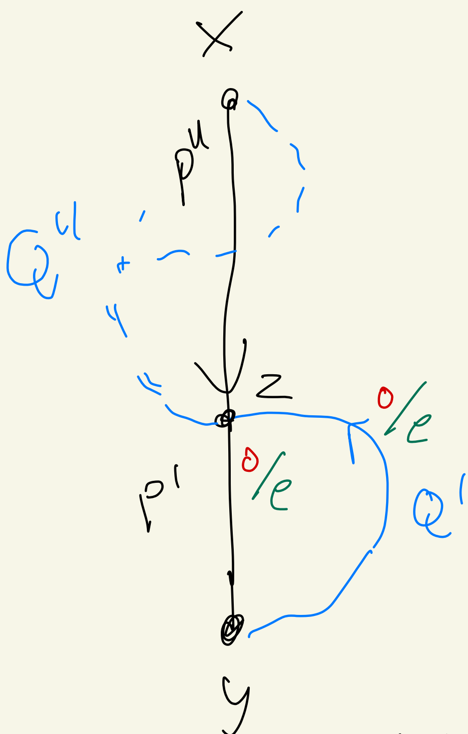
and $|A(P)| + |A(Q)|$ minimum $(*)$



if $V(P) \cap V(Q) = \{x, y\}$

odd cycle

↯



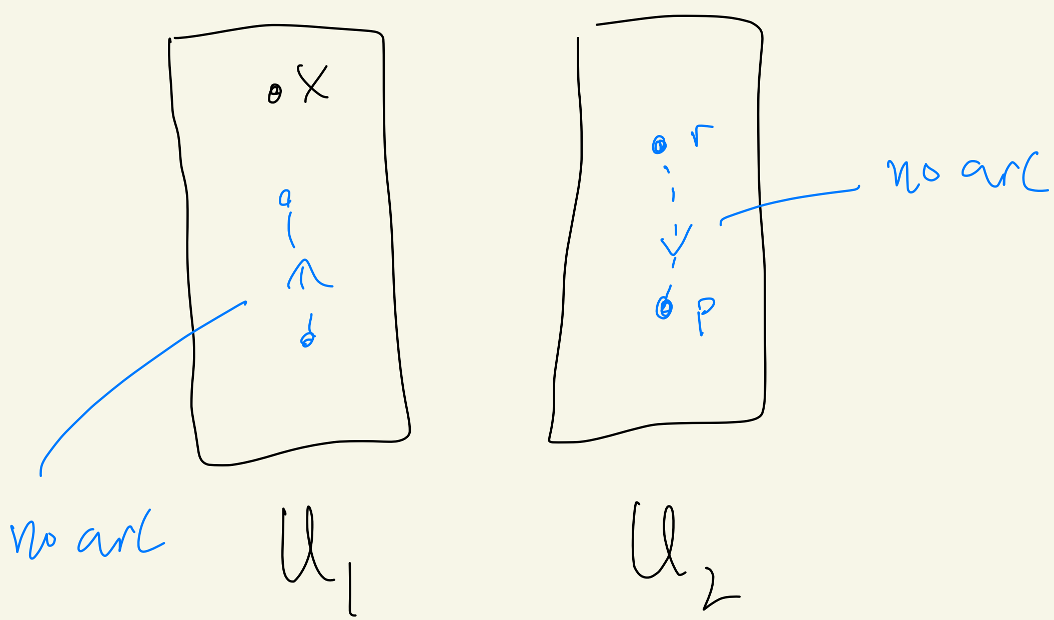
by (*) $|A(P')| \equiv |A(Q')| \bmod 2$

$\Rightarrow \quad |A(P'')| \not\equiv |A(Q'')| \bmod 2$ ↯

define

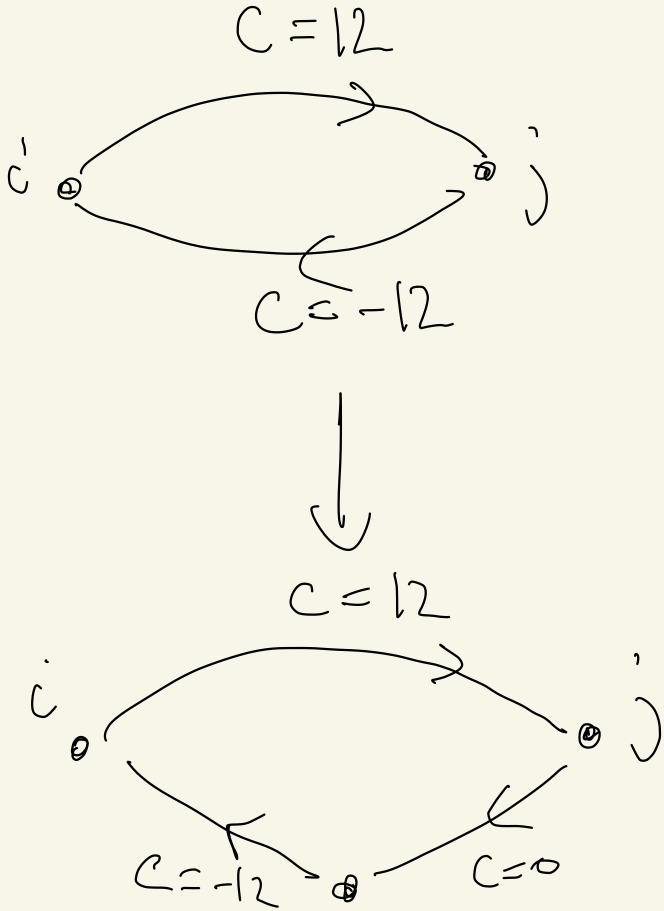
$U_1 = \{ y \mid \text{Every } (x,y)\text{-path is even} \}$

$U_2 = \{ y \mid \text{Every } (x,y)\text{-path is odd} \}$

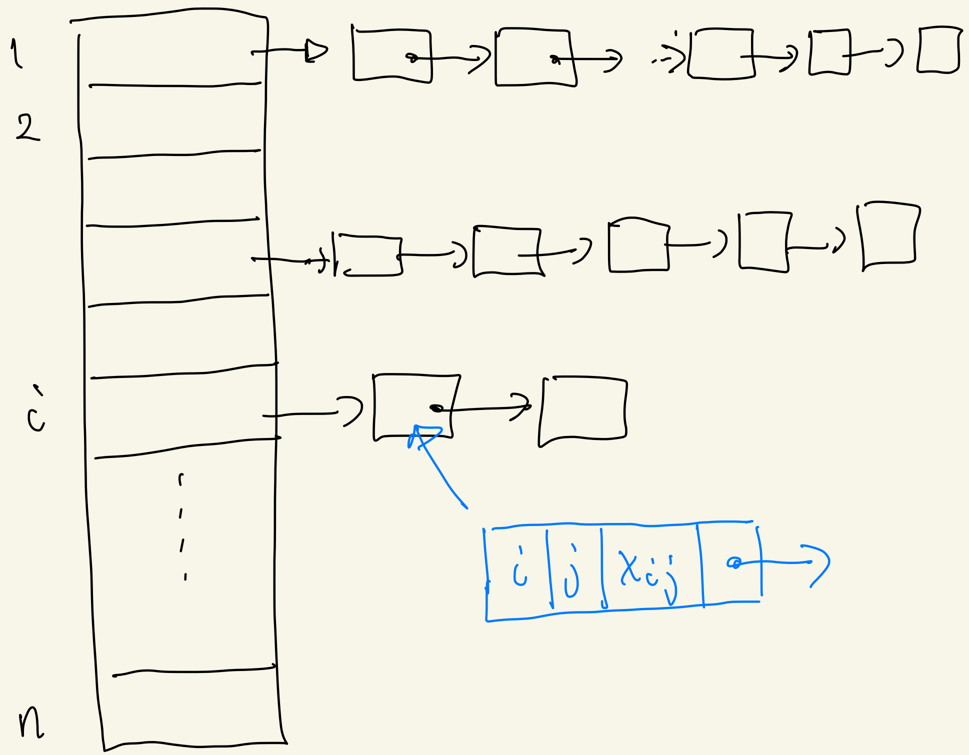


$\Rightarrow$ D is bipartite.

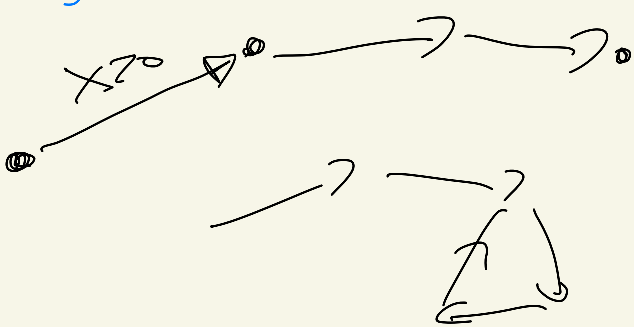
BJG 3.2



BJG 3.7



adjacency lists



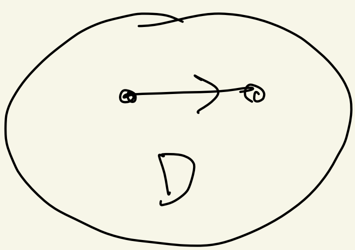
BJG 3.8

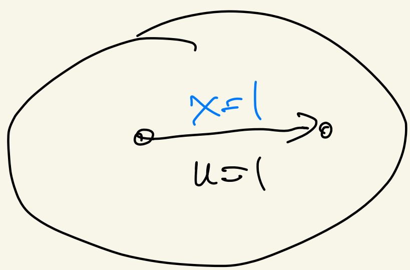
D is eulerian

$\Updownarrow$ def

$d^+(v) = d^-(v) \quad \forall v$

Show A(D) (arcs of D) can be decomposed into arc-disjoint cycles





$N_D = (V, A, \ell \equiv 0, u \equiv 1)$

x is a circulation

$\Rightarrow$ x decomposes in cycle flows

$\Rightarrow$ arc-disj cycles $W_1, W_2, \dots W_r$