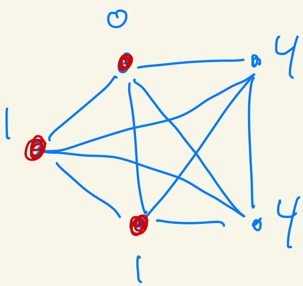
$G \longrightarrow D$

$i \,\text{—}\, j \qquad i \to j$ or $i \leftarrow j$

Problem Given $G=(V,E)$ $V=\{1,2,\dots,n\}$
and $a_1, a_2 \dots a_n$ $\sum_{i=1}^{n} a_i = |E|$
Does there exist an orientation D of G
such that $d_D^+(i) = a_i$?

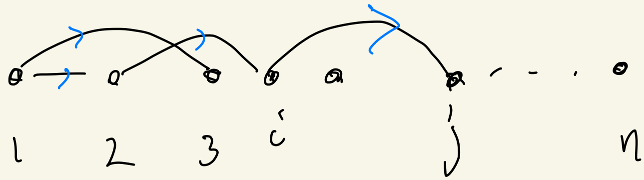
a_i arcs out



no orientatio

Step 1 $\longrightarrow$ D

G $\downarrow$ $D=(V,A)$

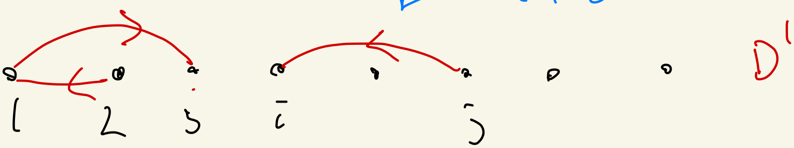


Suppose D' is a good orientation of G

$\left(d^+_{D'}(i) = a_i \quad \forall i \in [n] \right)$

Compare D' and D:

$\longrightarrow$ in D



interpret a $\{0,1\}$-flow x in $N=(V, A, l\equiv 0, u\equiv 1)$

by
$$x_{ij} = \begin{cases} 0 & \text{if we keep orientation of } ij \\ 1 & \text{if we want to reverse orientation of } ij \end{cases}$$

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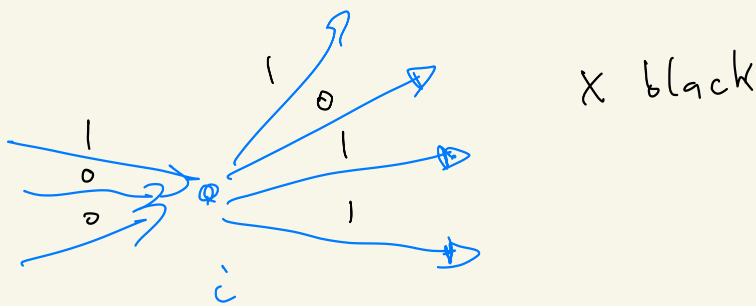
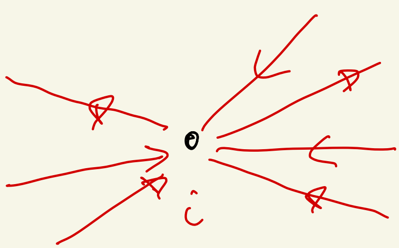
If we are given a $\{0,1\}$-flow x in N. Then resulting out-degree of i





$D' = D(x)$

$$d^+_{D'}(i) = d^+_D(i) - \sum_{ij\in A} x_{ij} + \sum_{ji\in A} x_{ji} \quad (*)$$

we want $d^+_{D'}(i) = a_i \quad \forall i \in [n]$

so $(*)$ gives us

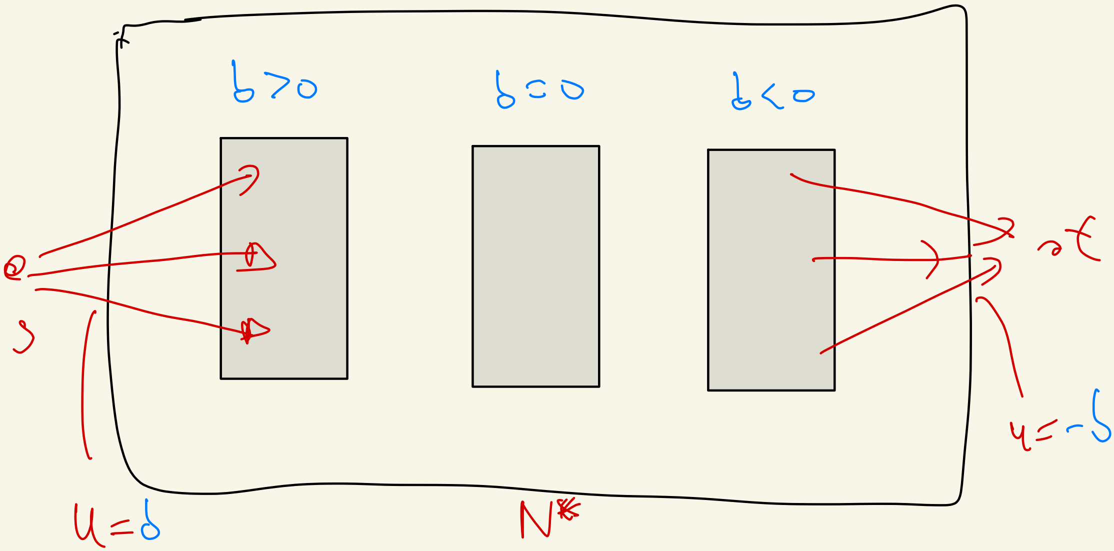
$$a_i = d^+_D(i) - b_x(i) \qquad \forall i\in[n]$$

$\Downarrow$

$$\boxed{\begin{array}{c} b_x(i) = d^+_D(i) - a_i = b(i) \quad \forall i \in [n] \\ 0 \le x_{ij} \le 1 \qquad \forall ij \in A \end{array}}$$

$$\sum_{i=1}^n b_x(i) = \sum_{i=1}^n d^+_D(i) - \sum_{i=1}^n a_i$$

$$|E| \quad - |E| \quad = 0$$



Then exists a feasible flow in N

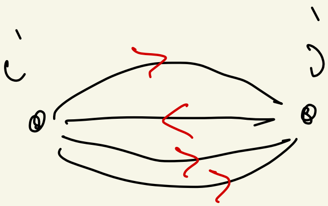
$\Updownarrow$

value of max flow in N^* is

$$\sum_{b(i)>0} b(i)$$

Ahuja application 1.3

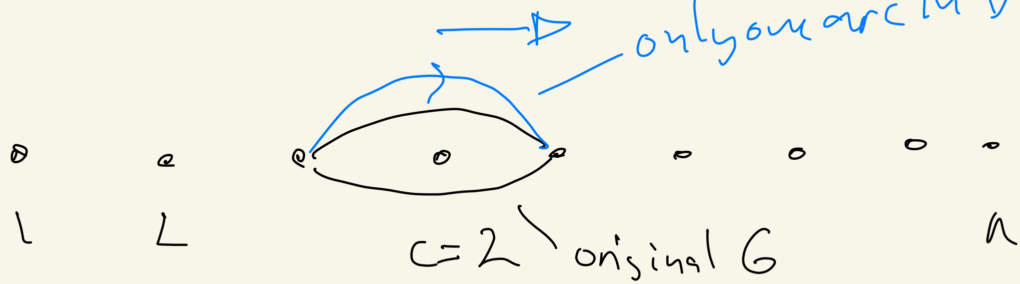
Round-Robin Tournament with $c \geq 1$ games per pair of teams. No draw



$c = 4$

Problem given $\alpha_1, \alpha_2 \cdots \alpha_n$

s.t $\sum_{i=1}^{n} \alpha_i = c\binom{n}{2}$

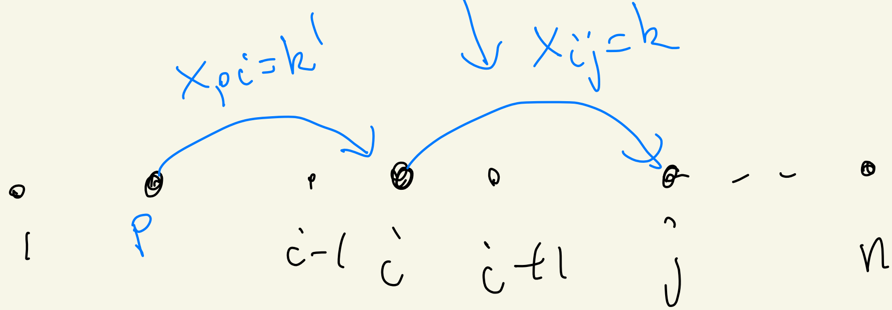


Interpret $x_{ij} = k \le C$ $\quad i<j$

as keeping k of cares

from i to j.

$$0 \le x_{ij} \le C \quad \forall\, i,j \quad i<j$$



number of wins for team i:

$$\sum_{ij\in A} x_{ij} + \sum_{pi\in A}(C - x_{pi})$$

$$= \sum_{ij\in A} x_{ij} - \sum_{pi\in A} x_{pi} + (i-1)C$$

$$\sum_{ij\in A} x_{ij} + \sum_{pi\in A} (c - x_{pi})$$

$$= \sum_{ij\in A} x_{ij} - \sum_{pi\in A} x_{pi} + (i-1)c \quad (*)$$

we want this to be α_i

so $(*)$ says we want

$$\delta_x(i) + (i-1)c = \alpha_i$$

$\Downarrow$

$$\delta_x(i) = \alpha_i - c(i-1) \quad \forall i.$$