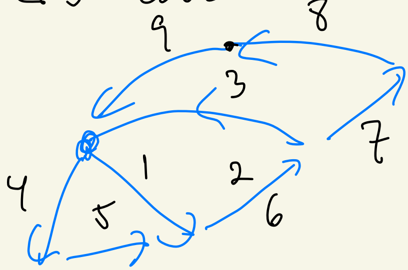
BJG 3.11.2

Chinese postman problem:

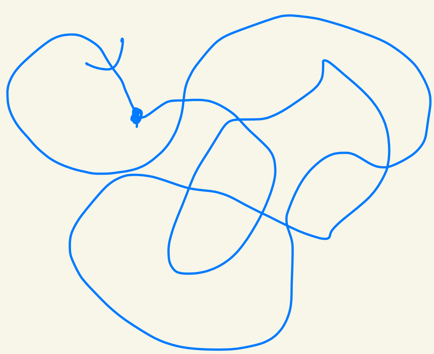
Given a digraph $D = (V, A)$

and $: A \to \mathbb{R}_0$

Find a minimum cost closed walk that covers all arcs



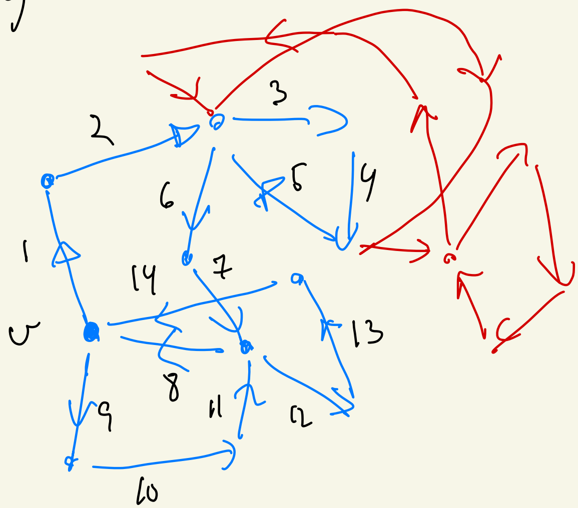
min cost



must cover each arc ≥ 1 time

Euler 1736 (for digraphs)

If $d^+(v) = d^-(v)$ $\forall v \in V$
and D is strongly connected
then we can all arcs precisely
once by a closed walk



Observation: the solution is an Eulerian super digraph of D (obtained by adding copies of arcs in D)

So we must decide how many copies of each arc ij we will use

Let $w_{ij} \geq 1$ be # of copies of ij for each $ij \in A$.

Then the tour costs $\sum_{ij \in A} c_{ij} w_{ij}$ (*)

want to minimize (*)

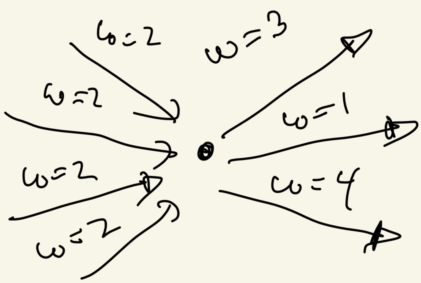
let $N = (V, A, \ell \equiv 1, u \equiv \infty, c)$

Thm 3.11.4 The cost of a min cost circulation in N is equal to min cost of chinese postman walk in D

P: if W is a solution in D and w_{ij} = # times ij is used on W

let $x_{ij} = w_{ij} \quad \forall ij \in A$

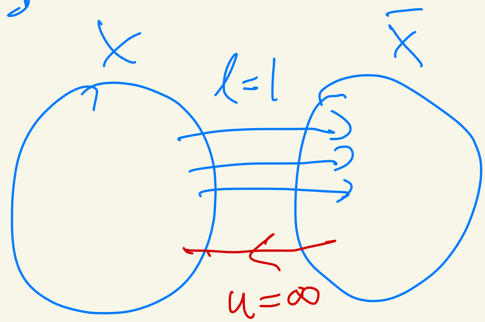
x is a circulation since W is a closed walk.



cost x
= cost W

Suppose x is a feasible integer circulation in N

always exists as D is strong



Let $D' = (V, A')$ be obtained by taking x_{ij} copies of ij $\forall ij \in A$

Then D' is eulerian $d^-_{D'}(v) = d^+_{D'}(v) \; \forall v$

Let $W \leftarrow$ euler tour of D'

$\text{cost } W = \text{cost } x$

3.11.3 subdigraphs with specified degrees.

Given $D=(V,A)$ $V=\{1,2,\dots,n\}$

and $a_1, a_2 \dots a_n$, $b_1, b_2 \dots b_n$

s.t $d^+(i) \geq a_i$ $d^-(i) \geq b_i$

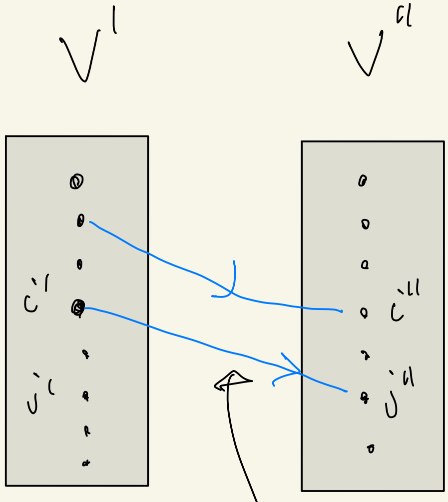
$$\sum_i a_i = \sum_j b_j = M \text{ for some } M \in \mathbb{Z}$$

Does there exist a subdigraph $D'=(V,A')$ of D with

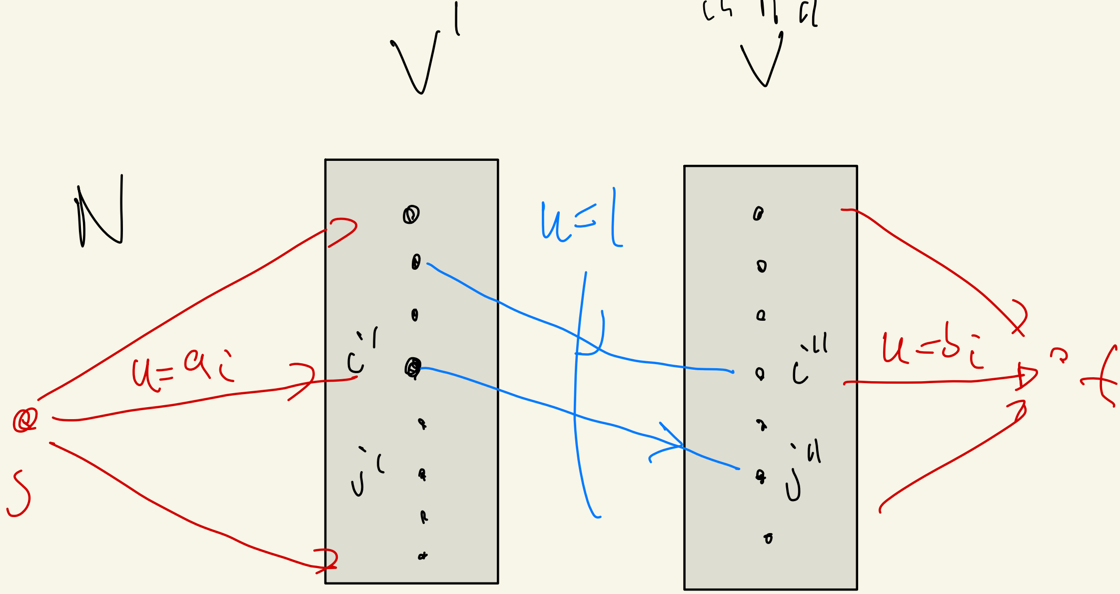
$$d^+_{D'}(i) = a_i$$

$$\forall i \in [n]$$

$$d^-_{D'}(i) = b_i$$



if $ij \in A$ then $i' \to j''$ in A'

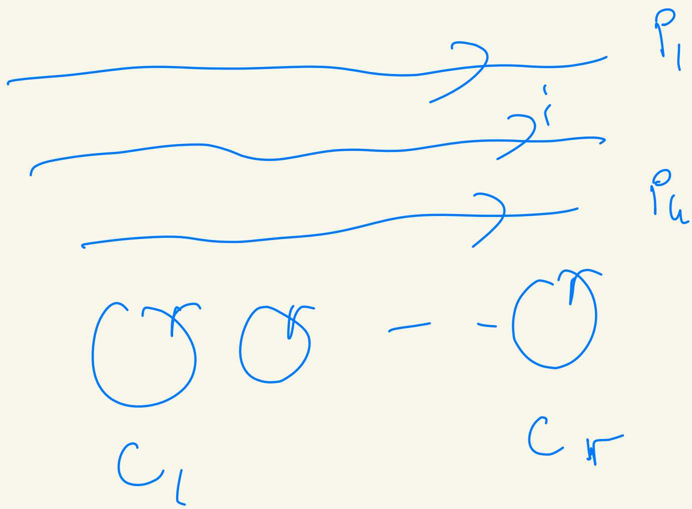


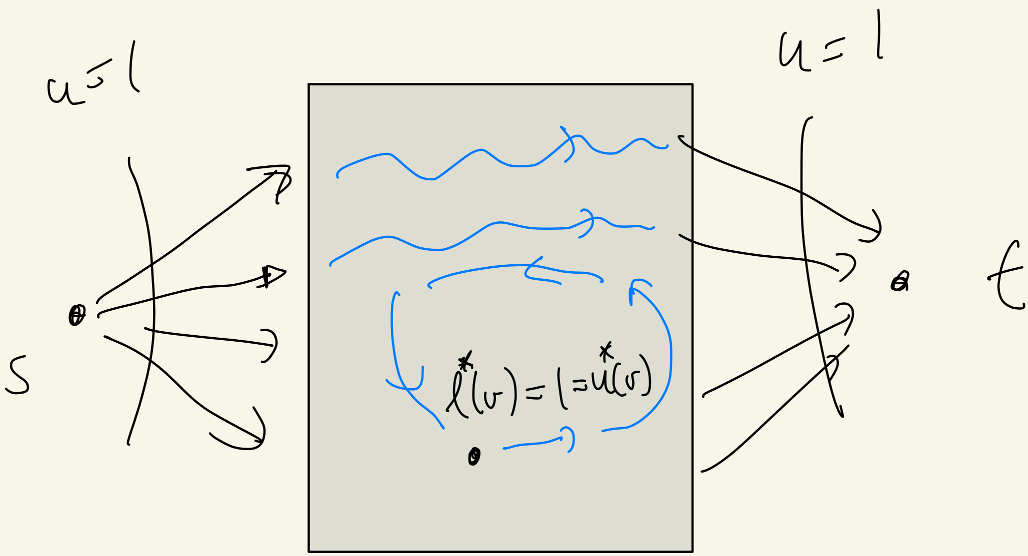
<u>claim</u> N has an (s,t)-flow of value M

$\Updownarrow$

D' exists

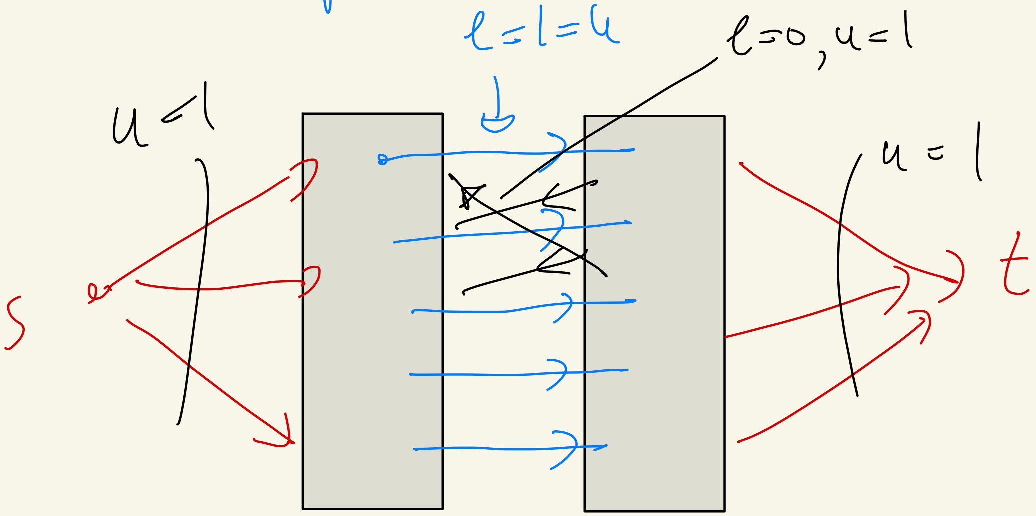
3.11.4 Path cycle factors

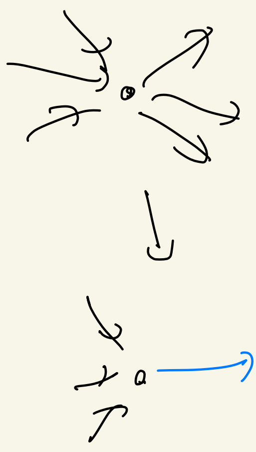




N_D: D

We can find min # of paths in a path-cycle-factor in D by solving a minimum value flow problem in N_D

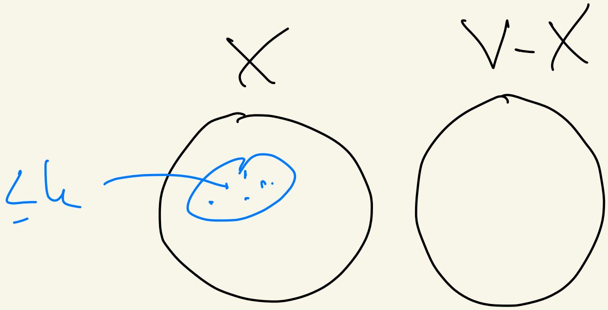


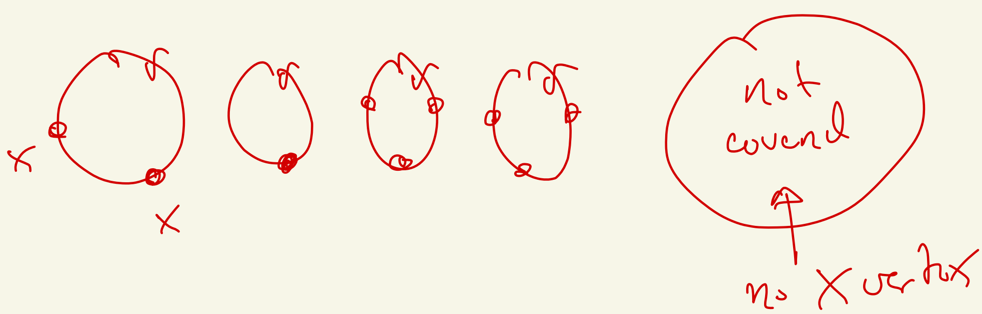


$O(\sqrt{nm})$

3.11.5 Cycle subdigraphs covering specified vertices

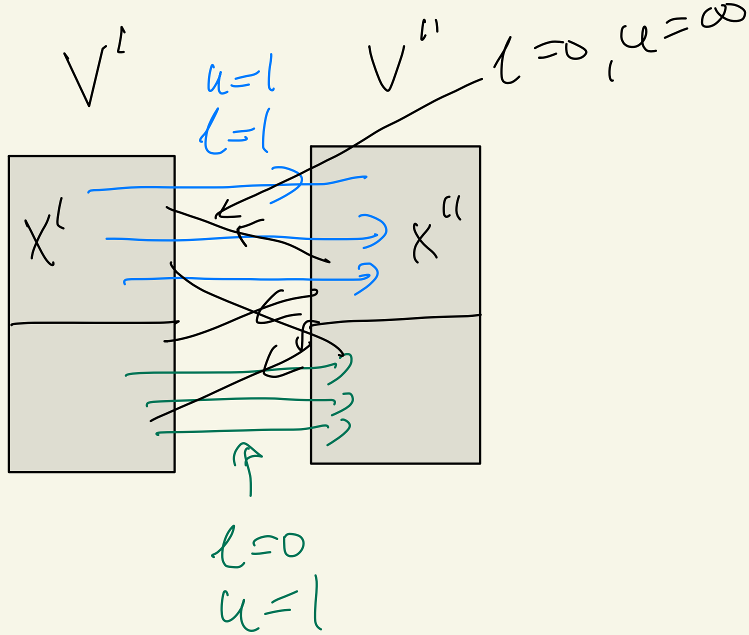
Thm 3.11.13 Let D be a k-strong digraph and suppose $\alpha(D[X]) \leq k$. Then D has a cycle subdigraph which covers X





Thm 3.11.13 Let D be a k-strong digraph and suppose $\alpha(D[X]) \leq k$. Then D has a cycle subdigraph which covers X

N:

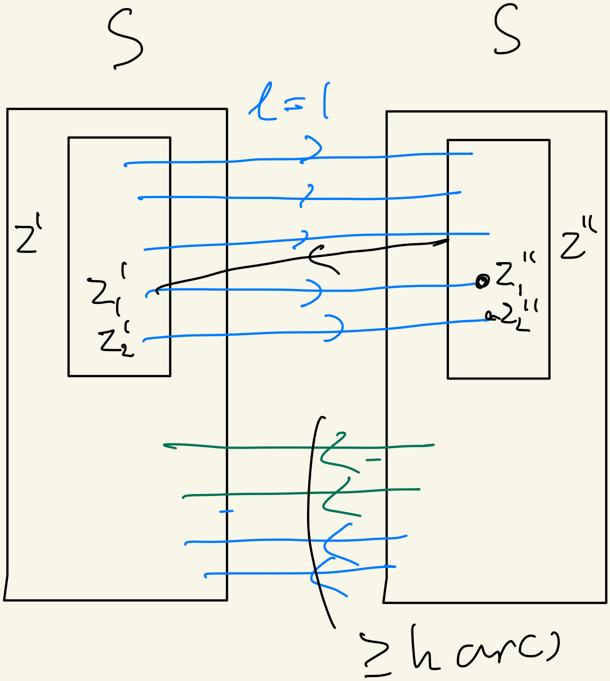


D has a cycle subdigraph covering X

$\Updownarrow$ claim

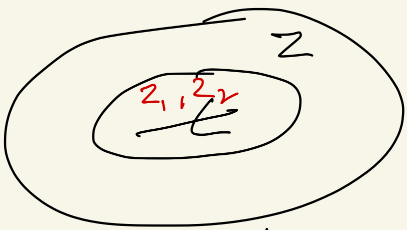
N has a feasible circulation

Suppose N does not have a feasible circulation. Then Hoffman:

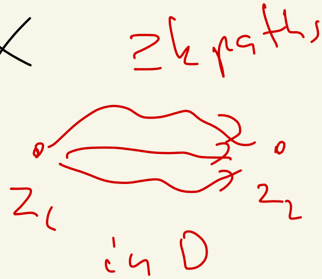


$\ell(S, \bar{S}) > u(\bar{S}, S)$ (*)

no $Z'' \to Z'$ arc.



X



- Z is independent

$$\ell(S, \bar{S}) \leq k \leq u(\bar{S}, S)$$

(*)