DM817 –Fall 2022 Weekly Note 2

Publication date for weekly notes

In the future I will try to publish to weekly notes on Thursday afternoons or Friday mornings.

Stuff covered in week 36

These correspond to the video lectures 1-4 as well as the videos labelled Exercises 1A and 1B.

- I discussed flows in the most general form where we are given a network $\mathcal{N} = (V, A, \ell, u, b, c)$ and seek a flow x which is feasible, that is $\ell_{ij} \leq x_{ij} \leq u_{ij}$ for every arcs $ij \in A$ and $b_x(v) = b(v)$ for every vertex $v \in V$.

- Then I showed that this very general model is equivalent in modelling power to the following models:

    - flows in networks, where there are no lower bounds on the arcs,

    - (s,t)-flows, where we seek a flow whose balance is non-zero only at two vertices s, t

    - circulations which are flows where the balance is zero everywhere. Now some arcs may have lowers bounds again, but we need only one such arc to have the full modelling power of the full model.

- I introduced the residual network $\mathcal{N}(x)$ with respect to a flow x in the network $\mathcal{N}$ and showed that this contains all the information we need to obtain any feasible flow (in particular any optimal flow) from a given feasible flow x. Notice that the definition of the residual network is different in the two books (Ahuja does not treat lower bounds). We shall use the definition from BJG (which coincides with Ahuja when lower bounds are zero).

- Using the residual network as an important tool I derived the max-flow-min-cut theorem which, despite its simplicity, plays a very important role in the theory as well as applications of flows.

- I proved the integrality theorem for flows which is a very important property of flows.

- I showed how to solve all the exercises on Weekly note 1, except Ahuja 2.51 which I forgot.

Activities in Week 37:

I will upload several shorter videos. They will cover

- Hoffman's circulation theorem. BJG 3.8.

- Minimum value flows. BJG 3.9.

- Application 6.6. from Ahuja Section 6.2.

- Ahuja 6.6 (corresponds to BJG 7.1.1 and 7.3.3 page 353)

- BJG 3.11.1

Plan for the physical meetings on September 15 and 16:

- I will show how to solve a few of the exercises from weekly note 1. If (after seing my videos on the exercises) you would like to see a solution of a particular exercise in class, please send me an email or write it on the discussion board via BB.

- I will cover selected parts from the videos about the topics above. Again you are welcome to specify things for which you want more explanation.

- We will take a look at some of the exercises below. The rest will be covered by a video.

Problems for Week 37:

- Ahuja 2.51, 3.48, 3.53, 3.54, 4.37
- BJG: 3.11, 3.15, 3.16, 3.18, 3.24, 3.28(a)+(b), 3.31