Alternative formulations

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Uncapacitated Facility Location (UFL)

Given a set of *potential* depots $N = \{1, 2, \ldots, n\}$ and a set $M = \{1, 2, \ldots, m\}$ of clients, suppose there is a fixed cost f_j associated with the use of depot j, and a transportation cost c_{ij} if all of client i's order is delivered from depot j.

The problem is to decide which depots to open, and which depots serves each client so as to minimize the sum of fixed and transportation cost.

Uncapacitated Lot-sizing (ULS)

The problem is to decide on a production plan for an n-period horizon for a single product. The basic model can be viewed as having data:

- f_t is the fixed cost of producing in period t.
- p_t is the unit production cost in period t.
- h_t is the unit storage cost in period t.
- d_t is the demand in period t.

- A subset of R^n described by a finite set of linear constraints $P = \{x \in R^n : Ax \leq b\}$ is a **polyhedron**.
- A polyhedron $P \subset R^{n+p}$ is a **formulation** for a set $X \subset Z^n \times R^p$ if and only if $X = (Z^n \times R^p) \cap P$.

◇ Given a set $x \subset R^n$ the **convex hull of** X, denoted conv(X) is defined as:

$$\text{conv}(X) = \{ \quad x : x = \sum_{i=1}^{t} \lambda_i x^i, \sum_{i=1}^{t} \lambda_i = 1, \lambda \geq 0 \text{ for } i = 1, \dots, t \text{ over all finite subsets } \{x^1, x^2, \dots, x^t\} \text{ of } X\}$$

◇ **Proposition:** conv(X) is a polyhedron

◇ **Proposition:** The extreme points of conv(X) all lie in X.

Ideal formulation

The ideal formulation in most cases consists of an enormous (exponential) number of inequalities needed to describe conv(X), and there is no simple characterization of them.

Instead we could rather ask:

Given two formulations P_1 and P_2 for X when can we say that one is better than the other?

Given a set $X \subset R^n$ and two formulations P_1 and P_2 for X, P_1 is a **better formulation** than P_2 if $P_1 \subset P_2$.

Projection

- first formulation: $\min\{cx : x \in P \cap Z^n\}$ with $P \subset R^n$.
- second formulation: $\min\{cx : (x, w) \in Q \cap (Z^n \times R^p)\}$ with $Q \subset R^n \times R^p$.
- Given a polyhedron $Q \subset R^n \times R^p$ the **projection of** Q onto the subspace R^n, denoted $\text{proj}_x Q$ is defined as:

$$\text{proj}_x Q = \{x \in R^n : (x, w) \in Q \text{ for some } w \in R^p\}$$