Curriculum for oral exam in DM85 Networks and Integer Programming, Spring 2007

Jørgen Bang-Jensen

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Teaching Material:

- L.A. Wolsey (IP): "Integer Programming", John Wiley, 1998
- J. Bang-Jensen and G. Gutin (BG): "Digraphs: Theory, Algorithms and Applications", Springer Verlag London 2001. Chapter 3 pages 95-122, 125-142, 147-158 (**flows, definitions and algorithms**)
- F.S. Hillier og G. J. Lieberman (HL): "Introduction to Operations Research", McGraw Hill 2001 (pages 468 - 485 and 492 - 502) (**PERT/CPM**)
- C.H. Papadimitriou and K. Steiglitz (PS): "Combinatorial Optimization", Prentice Hall 1982. Chapter 5. Pages 104-114. (**the primal-dual algorithm**).
- J. Bang-Jensen and B. Toft (BT): Handwritten notes on the primal-dual algorithm for the assignment and transportation problems.
- W. Cook, W. Cunningham, W. Pulleyblank and A. Schrijver (CCPS): "Combinatorial Optimization", John Wiley 1998. Chapter 7. Pages 241-271.
- T. Takkula, "Overview over optimization models in transportation", Course notes from Chalmers University Göteborg 2001. 14 pages.
- Used overheads (available on the home page of the course).

Curriculum:

- IP:
  - Chapters 1, 2, 3 (excl. sec. 3.6), 5, 7 (general knowledge of branch and bound as you have from DM19), 8 (excl sec. 8.7 and 8.8), 9 (excl. sec 9.4) 10, 11.1-11.4, 13.4
- BG:
  - pages 95-122, 125-142, 147-158
- HL:
  - pages 468 - 485 and 492 - 502.
- PS:
  - Pages 104-114.
- CCPS: Pages 261-265 (including Comb inequalities and their derivation)
- (BT): Handwritten notes on the primal-dual algorithm for the assignment and transportation problems.

Networks and Integer Programming, Spring 2006. Exam questions and og guide for the oral examination.

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The oral exam consists of an examination taking 25 minutes, to which each student has a preparation time of 30 minutes. Approx. the initial 15 minutes of the examination is at the disposal of the student, who presents a particular aspect of his/her question. The question is drawn by the student. In the following 10 minutes supplementary questions are asked by the examiner and censor - the topic range for these questions is the complete pensum.

In connection with the methods it is expected that the student can argue for the correctness, i.e. indicate why these produce the results claimed. A stringent mathematical proof is not expected, but of course it is permitted to give such a proof.

Below, the exam questions are stated. For each question, some keywords are given. These may form the basis for the presentation, however, the student is perfectly free to choose another approach for the presentation.

1. Formulation of Integer Programs
   - Examples of formulations
   - Definition of the term "formulation"
   - Good formulations
   - Comparing formulations
   - Example
2. Bounds in Integer Programming
   - Bounds as tools in solving problems
   - The concepts of primal and dual bounds
   - relaxation vs. duality
   - Examples of primal bounds
   - Examples of dual bounds
3. Project planning - Time-Cost Trade-off.
   - Basic concepts and problem formulation
   - Finding the project duration, a critical path and earliest (latest) start and finishing times for each activity.
   - Shortening of the project duration (crashing) formulated as an LP.
4. Flows: Basic concepts and the maximum (s,t)-flow problem
   - Basic concepts and problem formulation
   - Relations between various flow models (e.g. lower bounds, bounds on vertices, circulations etc).

- The residual network and its use to compare diffent flows.
- The flow augmenting path algorithm
- The Max Flow - Min Cut - theorem
- Correctness argument
- Reoptimizing a maximum flow after a change on one arc.

5. Circulations and minimum value (s, t)-flows

   - Hoffman's circulation theorem and the minimum flow - maximum demand theorem
   - Algorithms for finding a feasible circulation
   - Algorithms for finding a minimum value feasible (s, t)-flow
   - Applications of minimum value flows to scheduling

6. Max Flow: polynomial algorithms.

   - Ford-Fulkersons algorithm and argument that it is not polynomial
   - The Preflow-Push - algorithm
   - Correctness argument for Preflow-Push algorithm
   - Comparison of Preflow-Push algorithm with the flow augmenting path algorithm
   - Polynomial algorithms based on finding blocking flows in layered networks (Edmonds-Karp, Dinic, MKM (exercise 3.25).

7. Min Cost Flow: The cycle cancelling method and the build- up method.

   - Basic concepts and problem formulation
   - Proof of optimality criterion for minimum cost flows.
   - Complexity considerations (are the algorithms polynomial?).
   - Correctness argument for cycle cancelling and build-up.
   - Example of an iteration
   - Applications of minimum cost flows.
   - optimum tree solutions to minimum cost flow problems.

8. The Primal-Dual method for the Assignment and Transportation problems

   - Bipartite matching as a flow problem
   - The primal-dual algorithm specialized to the Assignment and the Transportation problem.
   - Updating dual variables.
   - Complexity issues (we can re-use edges with positive flow in next iteration).
   - A primal-dual algorithm for general minimum cost flows (Exercise 3.83)

9. Branch-and-Bound for STSP.
    - Basic components of Branch-and-Bound
    - TSP as a graph problem and as an 0-1 integer programming problem.
    - The 1-tree bound and its correctness
    - Description of a possible branching strategy
10. Branch-and-Cut for STSP.
    - Basic components for Branch-and-Cut
    - TSP as a graph problem and as an 0-1 integer programming problem
    - Families of valid inequalities for TSP (e.g. subtour elimination, and comb inequalities)
    - Branch-and-bound vs. Branch-and-Cut
11. Cutting Planes
    - The idea of cutting planes
    - Examples of "ad hoc" cuts
    - Derivation of the basic Gomory cut
    - Example
12. Lagrangean Relaxation (LR).
    - The relaxation idea
    - LP-relaxation for integer programming problems
    - Lagrange-relaxation
    - Pros and cons when using LR for calculation of dual bounds in Branch-and-Bound.
13. Column Generation
    - Basic idea in CG and Danzig- Wolfe reformulation of an IP
    - Solving the master linear program (describe process)
    - Solving STSP by column generation
    - How to use column generation together with B & B to solve 0-1 integer programs.
14. Uncapacitated Facility Location.
    - Basic concepts and problem formulation
    - Equivalent formulations
    - Lagrangean relaxation for UFL
    - An example of an UB calculation
15. 2-edge connectivity augmentation

- Problem formulation and a mathematical models for E2AUG and E1-2AUG
- The heuristic from Project 1.
- Solving the problems using cutting planes.
- Using flows to detect violated cuts in LP solution.
- The specializations E2AUG(S) and E1-2AUG(S)
- Special case of E1-2AUG(S) when $|S| = 2$.